Analysis of the driving potential of a solar chimney power plant

D.G. Kröger*   D. Blaine†

(*Received April 1999; Final version September 1999*)

In a solar chimney power plant energy is generated due to a stream of air that is heated in a solar collector and driven by buoyancy effects through a turbine located at the base of the chimney. The driving potential of this air stream is essentially the difference in pressure between the ambient atmosphere at ground level and the pressure of the heated air at the base of the tower. Since this pressure differential is small compared to the absolute values from which it is determined and since it can be significantly influenced by ambient conditions, detailed theoretical modelling incorporating adequate and reliable meteorological data is required. The merits of different theoretical models as well as the influence of prevailing ambient conditions on the driving potential are evaluated. It is shown that moist air generally improves the draft and that condensation may occur in the chimney under certain conditions.

Nomenclature

a	Constant
b	Constant
g	Gravitational acceleration, m/s^2
H	Height, m
i_{fgwo}	Latent heat of vaporization at 0°C, J/kg
p	Pressure, N/m^2
R	Gas constant, J/kg K
T	Temperature, K
w	Humidity ratio, kg water vapour/kg dry air
z	Altitude, m

Greek letters

γ	Ratio of specific heats, c_p/c_v
ρ	Density, kg/m^3

Subscript

a	Air or atmosphere
c	Condensation
s	Saturation
v	Vapour
w	Water

Introduction

A solar chimney power plant consists of a central chimney that is surrounded by a transparent canopy located a few metres above ground level as shown schematically in Figure 1. As a result of the greenhouse effect, the temperature of the ground under the canopy rises significantly, heating the air which due to buoyancy flows radially inwards towards the chimney. Energy may be extracted from this stream of air by means of a turbine located near the base of the chimney.[1]

The performance of such a solar chimney power plant is dependent on, among other things, the pressure differential created by the relatively buoyant or less dense warm air in the chimney and the corresponding column of denser ambient air. When evaluating this differential theoretically, it is important to model relevant atmospheric characteristics accurately. Small errors in the estimation of absolute relevant pressures can lead to significant errors in the determination of the relatively small differential that drives the system. The accuracy of any such analysis obviously depends to a large extent on the available meteorological data.

Analysis

Consider the solar power plant shown in Figure 1. The chimney has an effective height H. Specified ambient conditions near ground level outside the collector at 1 include the atmospheric pressure p_1, the corresponding temperature T_1, whilst the temperature of the heated air is T_2 at the base of the chimney.

With this very limited amount of information the driving potential or pressure differential between (1) and (2) (in the absence of any flow) is given approximately by

*Department of Mechanical Engineering, University of Stellenbosch, Private Bag X1, Matieland, 7602 South Africa

†Department of Mechanical Engineering, University of Stellenbosch

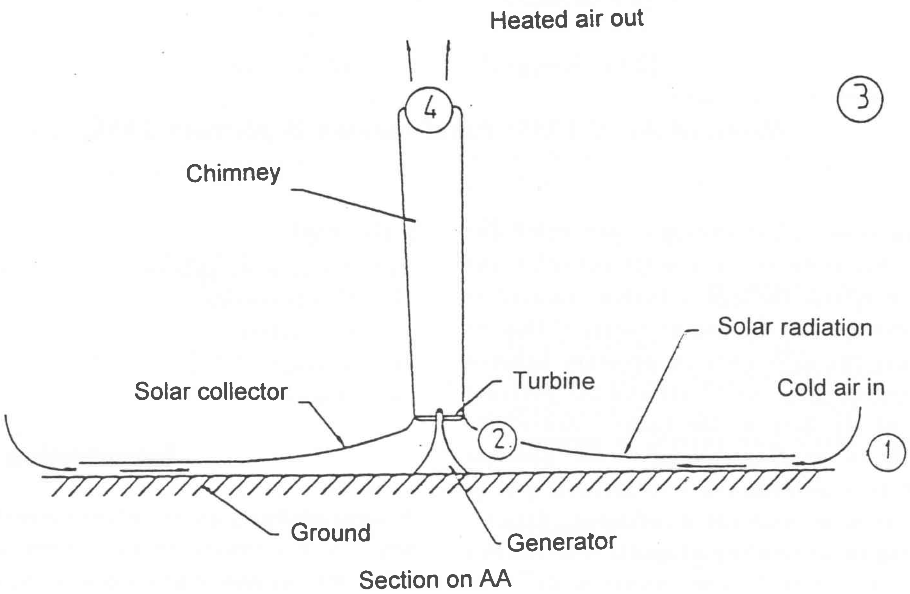

Figure 1 Schematic of solar chimney power plant

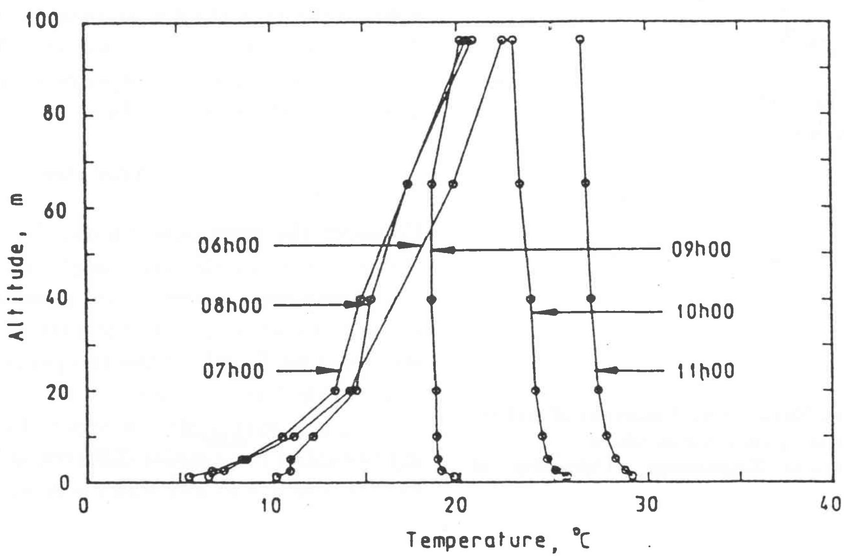

Figure 2 Hourly temperatures in °C at different elevations

$$p_1 - p_2 \approx \rho_1 g H - \rho_2 g H = (\rho_1 - \rho_2) g H \quad (1)$$

For dry air the perfect gas relation gives the density

$$\rho = p/RT \quad (2)$$

Substitute equation (2) into equation (1) and find with $p_2 \approx p_1$

$$p_1 - p_2 \approx p_1 g H (1/T_1 - 1/T_2)/R \quad (3)$$

In equation (1) it is implied that the density of the air inside and outside the chimney is constant or changes linearly with height. In practice this is not the case due to variations in pressure and temperature with altitude. An example of measured hourly temperature distributions near ground level in a relatively arid area is shown in Figure 2.[2] During most of the day the temperature distribution is close to the dry adiabatic lapse rate (DALR) of 0.00975 K/m. During the early morning hours temperature inversions are observed.

Inside the chimney the temperature change will also be according to the DALR. The latter is applicable when dry air experiences an isentropic change in state i.e.

$$p/\rho^{\gamma} = \text{constant} \quad (4)$$

Substitute equation (2) into equation (4) and differentiate with respect to altitude i.e.

$$\frac{(1-\gamma)\, dp}{\gamma p}\frac{dp}{dz} + \frac{1}{T}\frac{dT}{dz} = 0 \quad (5)$$

The pressure gradient in a gravity field is given by

$$dp/dz = -\rho g \quad (6)$$

Substitute equations (2) and (6) into equation (5) and find

$$\frac{dT}{dz} = \frac{g(1-\gamma)}{\gamma R} \quad (7)$$

For dry air with $\gamma = 1.4$, $R = 287.08$ J/ kg K and $g = 9.8$ m/s^2 find the DALR

$$\frac{dT}{dz} = \frac{9.8(1-1.4)}{1.4 \times 287.08} = -0.00975 \text{ K/m}$$

The ambient temperature distribution may therefore be obtained by integrating equation (7) with T_1 the temperature at $z = 0$ i.e.

$$T = T_1 + g(1-\gamma) z/(\gamma R) = T_1 - 0.00975\, z \quad (8)$$

According to equations (2) and (6) the corresponding pressure gradient is

$$dp/dz = -pg/(RT) \quad (9)$$

Substitute equation (8) into equation (9) and integrate to find the ambient pressure at an elevation z

$$p = p_1 (1 - 0.00975\, z/T_1)^{3.5} \quad (10)$$

The ambient pressure at an elevation corresponding to the outlet of the chimney is therefore

$$p_3 = p_1 (1 - 0.00975\, H/T_1)^{3.5} \quad (11)$$

The pressure at the outlet of the chimney is essentially equal to the ambient pressure at that elevation i.e. $p_4 = p_3$.

For a DALR inside the chimney

$$p_4 = p_2 (1 - 0.00975\, H/T_2)^{3.5} \quad (12)$$

or

$$p_2 = p_4 (1 - 0.00975\, H/T_2)^{-3.5} \quad (13)$$

With $p_4 = p_3$ substitute equation (11) into equation (13) and find the pressure difference

$$p_1 - p_2 = p_1 \left[1 - \left\{ \begin{array}{c} (1 - 0.00975 H/T_1)/ \\ (1 - 0.00975 H/T_2) \end{array} \right\}^{3.5} \right] \quad (14)$$

Although the temperature lapse in arid areas is often close to the DALR, deviations may occur due to moisture in the air and the interaction between solar radiation and the atmosphere. An International Standard Atmosphere (ISA) intended to approximate the atmospheric conditions prevailing for most of the year in temperate latitudes is defined as having a mean sea level pressure of 101325 N/m^2, a corresponding temperature of 15°C with a mean lapse rate of 0.0065 K/m to a height of 11 km. According to this lapse, the ISA troposphere obeys a law of the form $p/\rho^{1.235}$ = constant.

With a temperature lapse rate of 0.0065 K/m and the pressure gradient as given by equation (9), find the pressure at any elevation for the ISA, i.e.

$$p = p_1 (1 - 0.0065 z/T_1)^{5.255} \quad (15)$$

If the ISA is assumed to prevail outside the chimney and the DALR prevails inside the chimney the driving pressure differential is given by

$$p_1 - p_2 = p_1 \left[1 - \begin{array}{c} (1 - 0.0065 H/T_1)^{5.255}/ \\ (1 - 0.00975 H/T_2)^{3.5} \end{array} \right] \quad (16)$$

The presence of water vapour in the air can significantly influence the draft in the solar power plant due to its effect on the density, which in such a case is given by

$$\rho_{av} = (1+w)\left[1 - w/(w+0.622)\right] p/RT \qquad (17)$$

Substitute this density into equation (4) and with γ replaced γ_{av}, differentiate with respect to altitude to find the temperature gradient

$$\frac{dT}{dz} = -\left(\frac{1-\gamma_{av}}{\gamma_{av}}\right)\frac{T}{p}\frac{dp}{dz} \qquad (18)$$

Substitute equations (6) and (17) into equation (18) and with $\gamma_{av} = c_{pav}/c_{vav}$ and $c_{pav} - c_{vav} = R_{av}$ find

$$\frac{dT}{dz} = -\frac{R_{av}\, g\,(1+w)\left[1 - w/(w+0.622)\right]}{c_{pav} R} \qquad (19)$$

Substitute $R_{av} = R + wR_v = R(1 + w\,R_v/R) = R(1 + 461.52w/287.08) = R(1 + w/0.622)$ and $c_{pav} = c_{pa} + wc_{pv} \approx c_{pa}(1 + 1.9w)$ into equation (18) and find

$$\frac{dT}{dz} = \frac{-g\,(1+w)}{c_{pa}\,(1+1.9w)} = \frac{-9.8\,(1+w)}{1006\,(1+1.9w)} = \frac{-0.00975\,(1+w)}{(1+1.9w)} \qquad (20)$$

Integrate equation (20) and find with $T = T_1$ at $z = 0$

$$T = T_1 - 0.00975\,(1+w)\,z/(1+1.9w) \qquad (21)$$

From equations (6), (17), and (21) find

$$\frac{1}{p}\frac{dp}{dz} = \frac{-g\,(1+w)\left[1 - w/(w+0.622)\right]}{R\left[T_1 - 0.00975\,(1+w)\,z/(1+1.9w)\right]} \qquad (22)$$

Integrate equation (22) to give the change in pressure inside the chimney.

$$p = p_2 \times \left[1 - \frac{0.00975\,(1+w)\,z}{(1+1.9w)\,T_2}\right]^{\frac{g\,(1+1.9w)}{0.00975R}\left[1-\frac{w}{(w+0.622)}\right]} = p_2\left[1 - \frac{0.00975\,(1+w)\,z}{(1+1.9w)\,T_2}\right]^{2.177(1+1.9w)/(w+0.622)} \qquad (23)$$

In the case where the temperature distribution outside the tower is according to the DALR and that inside the tower is given according to equation (20) the pressure differential is

$$p_1 - p_2 = p_1\left[1 - \left(1 - \frac{0.00975H}{T_1}\right)^{3.5} \Big/ \left\{1 - \frac{0.00975\,(1+w)\,H}{(1+1.9w)\,T_2}\right\}^{2.177(1+1.9w)/(w+0.622)}\right] \qquad (24)$$

If the humidity ratio of the air entering the chimney exceeds a certain value, condensation may occur with the result that a mist of droplets appears after some elevation in the chimney. The density of this air-vapour-water mixture is given by

$$\rho_{avw} = (1+w_s)\left[1 - w_s/(w_s+0.622)\right] \times p/RT + (w_{sc} - w_s)\,p_a/RT \approx \left[(1+w_s)\left\{1 - w_s/(w_s+0.622)\right\} + (w_{sc} - w_s)\right] p/RT \qquad (25)$$

if $p_{vs} \ll p$. The humidity ratio w_{sc} is that value of the ratio where condensation first occurs in the chimney.

With this density, the pressure gradient above the elevation where condensation first occurs is, according to equation (6)

$$dp/dz = -g\left[(1+w_s)\left\{1 - w_s/(w_s+0.622)\right\} + (w_{sc} - w_s)\right] p/RT \qquad (26)$$

According to Kröger[2] the temperature gradient in saturated air in a gravity field is given by

$$\frac{dT}{dz} = \frac{-(1+w_s)\,g}{(c_{pa} + w_s c_{pv}) + \left[\begin{array}{c} i_{fgwo} - (c_{pw} - c_{pv}) \\ \times\,(T - 273.15)\end{array}\right] dw_s/dT} \qquad (27)$$

The humidity ratio can be expressed as

$$w_s = 0.622\,p_{vs}/(p - p_{vs}) \qquad (28)$$

In the range 273.15 K < T < 313.15 K the saturation pressure of water vapour can be approximated by the relation

$$p_{vs} = 2.36874 \times 10^{11} \exp(-5406.1915/T) \qquad (29)$$

Differentiate equation (28) with respect to temperature and find

$$\frac{dw_s}{dT} = \frac{0.622\left[p\,(dp_{vs}/dT) - p_{vs}\,(dp/dT)\right]}{(p - p_{vs})^2} \qquad (30)$$

Upon differentiating equation (29) with respect to temperature find

$$dp_{vs}/dT = 5406.1915\, p_{vs}/T^2 \qquad (31)$$

Furthermore, with equations (6) and (25) find

$$\frac{dp}{dT} = \left(\frac{dp}{dz}\right)\left(\frac{dz}{dT}\right) = -\frac{\rho_{avw}\, g}{(dT/dz)} =$$

$$-\frac{[(1+w_s)\{1-w_s/(w_s+0.622)\}+(w_{sc}-w_s)]\, pg}{RT\,(dT/dz)} \qquad (32)$$

Substitute equations (31) and (32) into equation (30) and find with equations (28) and (29) that

$$\frac{dw_s}{dT} = \frac{0.67872\times 10^{-11} w_s^2 p \exp(5406.1915/T)}{T}\left[\frac{5406.1915}{T} + \frac{g\,[(1+w_s)\{1-w_s/(w_s+0.622)\}+(w_{sc}-w_s)]}{R(dT/dz)}\right] \qquad (33)$$

Substitute equation (33) into equation (27) and find for the case where $(w_{sc}-w_s) \ll (1+w_s)\{1-w_s/(w_s+0.622)\}$, that

$$\frac{dT}{dz} = \xi_T = \frac{-g\,(1+w_s)\,[1+0.42216\times 10^{-11} w_s^2 p \exp(5406.1915/T)\, i_e/\{(w_s+0.622)\,RT\}]}{(c_{pa}+w_s c_{pv}) + 3.6693\times 10^{-8} w_s^2 p \exp(5406.1915/T)\, i_e/T^2} \qquad (34)$$

where $i_e = i_{\text{fgwo}} - (c_{pw} - c_{pv})(T - 273.15)$

Analytical integration of this equation is not possible. Numerically it is however readily shown that the temperature gradient determined at a particular pressure and temperature hardly changes at higher elevations where both the pressure and temperature will be lower, i.e. the temperature gradient is approximately constant, or

$$T = T_{sc} + \xi_T z_s \qquad (35)$$

Substitute equation (35) into equation (26) and find for $(w_{sc}-w_s) \ll (1+w_s)\{1-w_s/(w_s+0.622)\}$ that

$$\frac{dp}{dz} = \frac{-0.622 g\,(1+w)\,p}{R\,(w_s+0.622)\,(T_{sc}+\xi_T z_s)} \qquad (36)$$

Upon integration of equation (36) find the approximate pressure above the elevation where the air initially becomes saturated and condensation commences.

$$p_s \approx p_{sc}\,[1+\xi_T z_s/T_{sc}]^{-0.622g(1+w_s)/[R\xi_T(w_s+0.622)]}$$

$$= p_{sc}\,[1+\xi_T z_s/T_{sc}]^{-0.021233g(1+w_s)/[\xi_T(w_s+0.622)]} \qquad (37)$$

Results and conclusions

Various models for determining the driving potential for the flow through a solar chimney power plant have been presented. To illustrate the dependence of these models on the parameters that influence the potential, a practical numerical example is presented.

Consider a solar power plant that has a chimney of height 1 500 m. The ambient air pressure $p_1 = 90\,000$ N/m^2 and the corresponding temperature is $T_1 = 30$°C (303.15 K). If the rise in air temperature in the solar collector is 20°C such that $T_2 = 50$°C (323.15 K) the driving potential as given by equation (3) is $(p_1 - p_2) = 940.9$ N/m^2. If it is assumed that the DALR is applicable both outside and inside the chimney, equation (14) gives $(p_1 - p_2) = 981.3$ N/m^2 whilst for an ISA outside, and a DALR inside the chimney, equation (16) gives $(p_1 - p_2) = 868.1$ N/m^2. In view of the measurable difference in the potentials given by these models it is obvious that great care must be taken in the evaluation of driving potentials.

If a DALR is assumed outside and inside the chimney for a given ambient temperature of $T_1 = 30$°C and a temperature rise of 20°C in the collector, the potential $(p_1 - p_2)$ will increase with increasing ambient pressure as shown in Figure 3. The change in potential for this chimney at a pressure of $p_1 = 90\,000$ N/m^2 and $T_2 - T_1 = 20$°C as a function of ambient temperature is shown in Figure 4. If the ambient pressure remains at 90 000 N/m^2 and $T_1 = 30$°C, the potential improves if the outlet temperature of the collector, T_2, increases as shown in Figure 5.

If the air in the chimney contains water vapour, the draft is generally improved. The potential for moist air is shown in Figure 6 for an ambient pressure of 90 000 N/m^2, $T_1 = 30$°C and $T_2 - T_1 = 20$°C. For a DALR external to the tower and unsaturated conditions inside the chimney, the potential is given by equation (24). A similar equation may be deduced for an ISA external to the chimney. The potential according to equation (3) is shown by a horizontal line. When the relative humidity

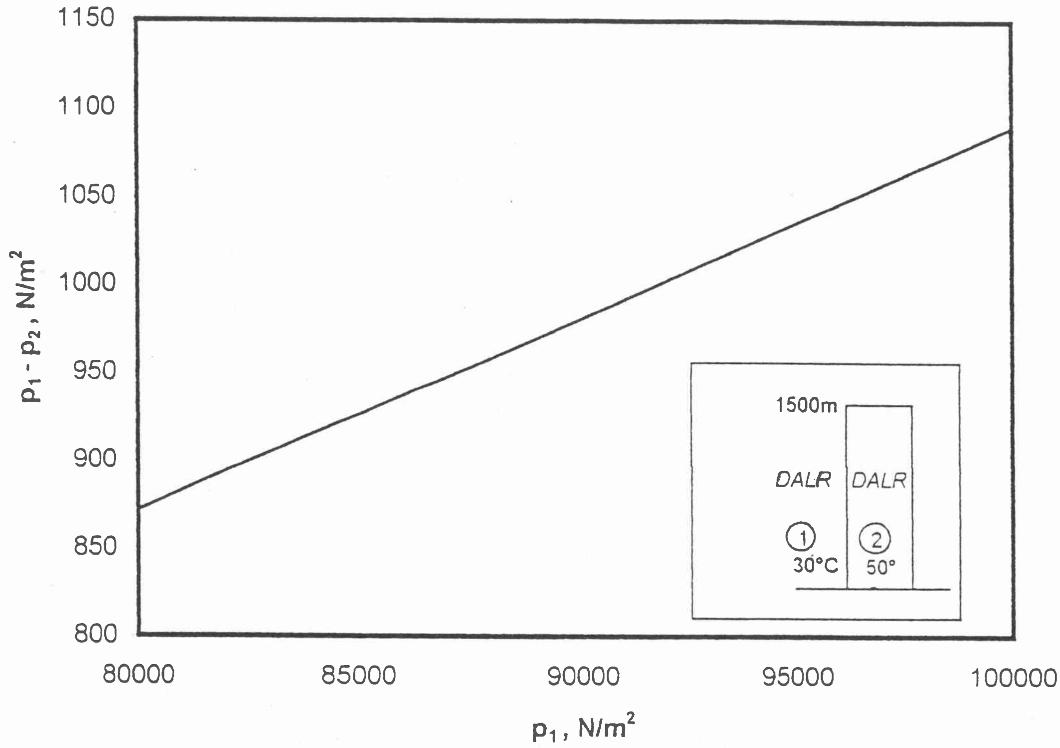

Figure 3 Potential as function of ambient pressure

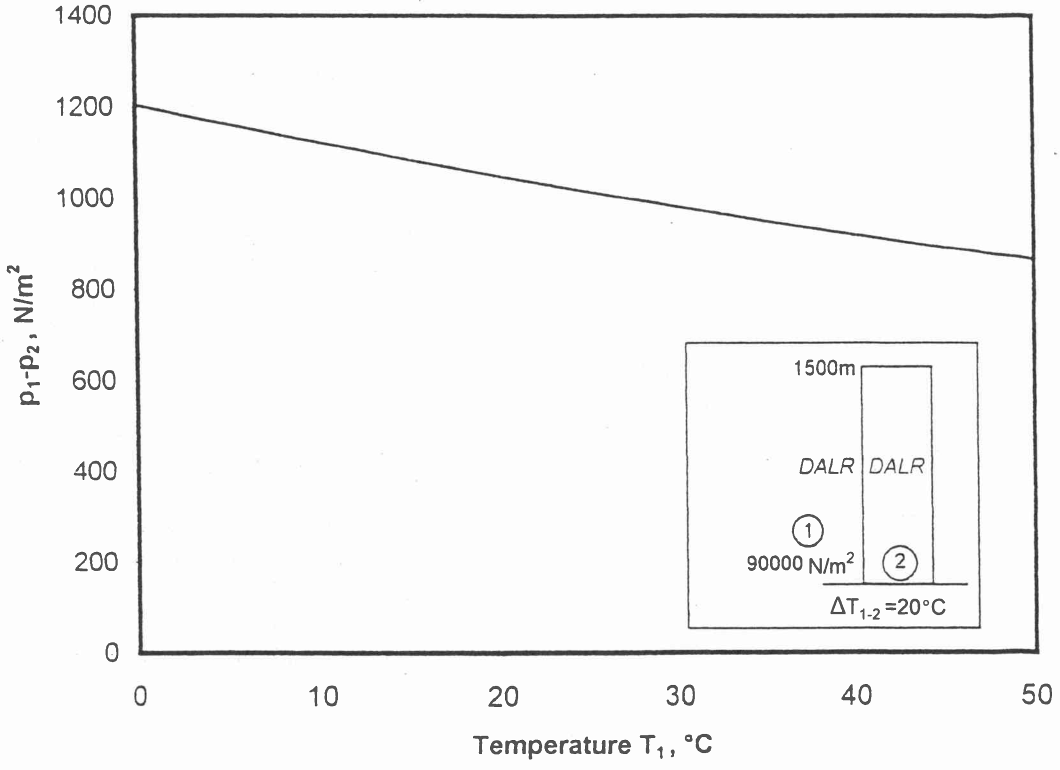

Figure 4 Potential as function of ambient temperature

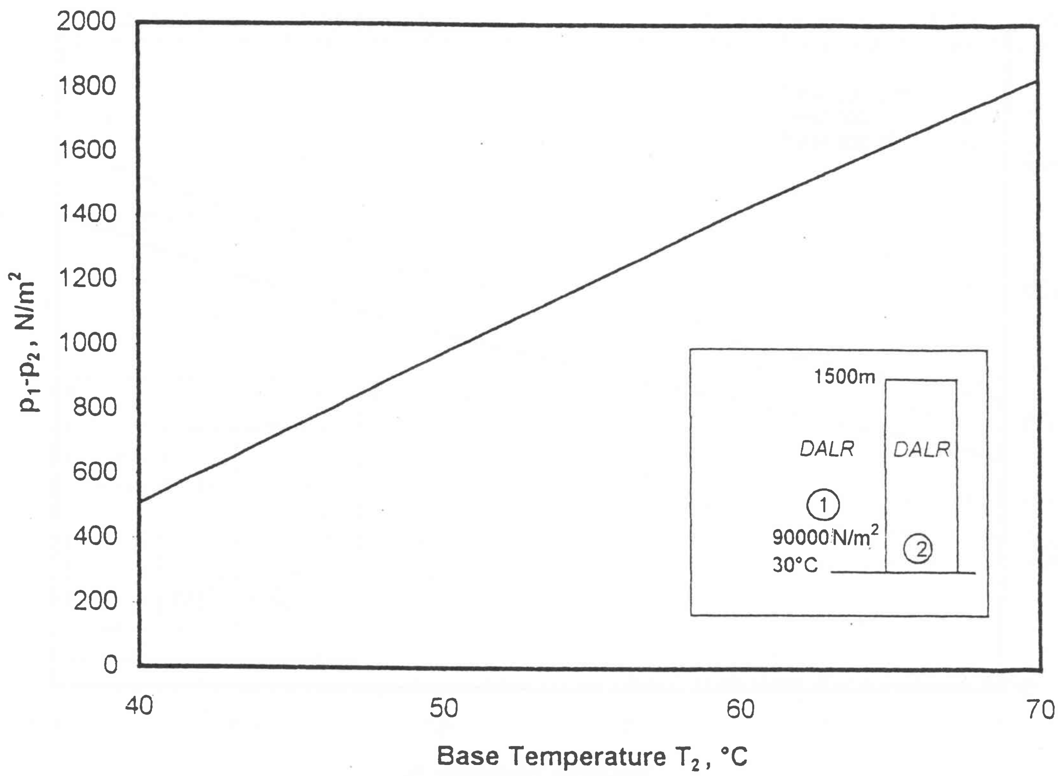

Figure 5 Potential as function of temperature rise in collector

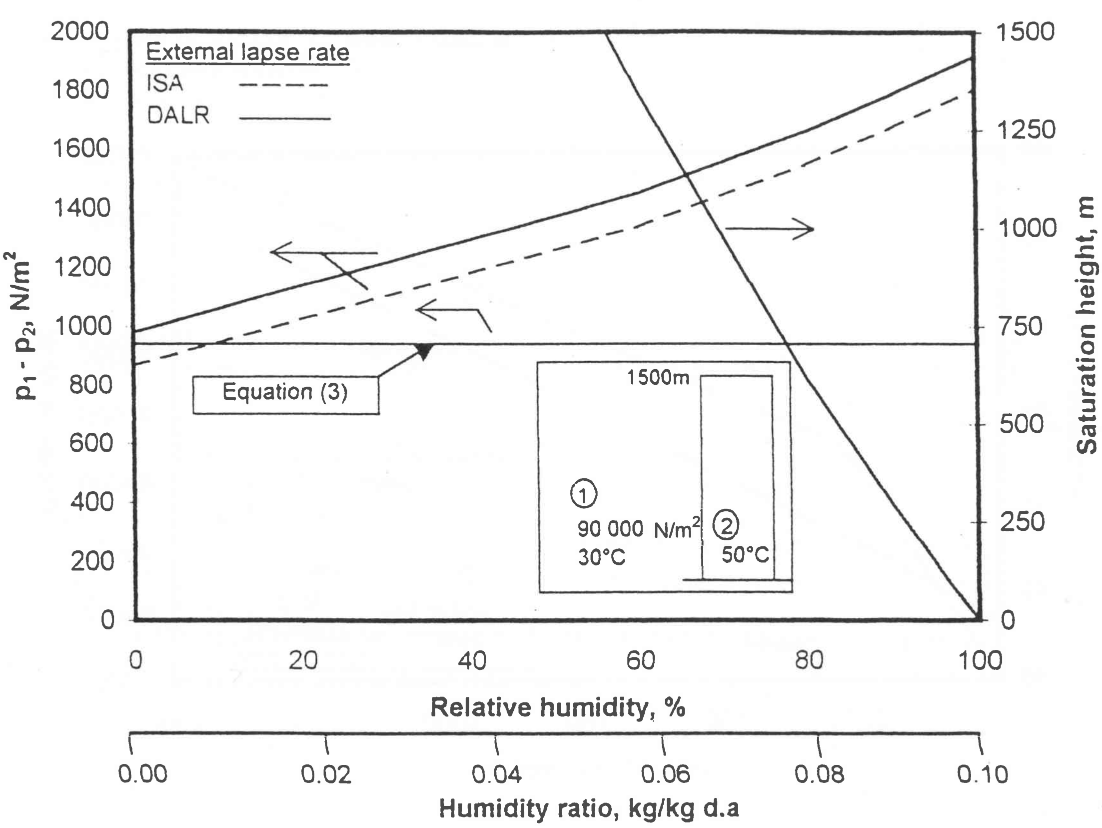

Figure 6 Potential as a function of humidity

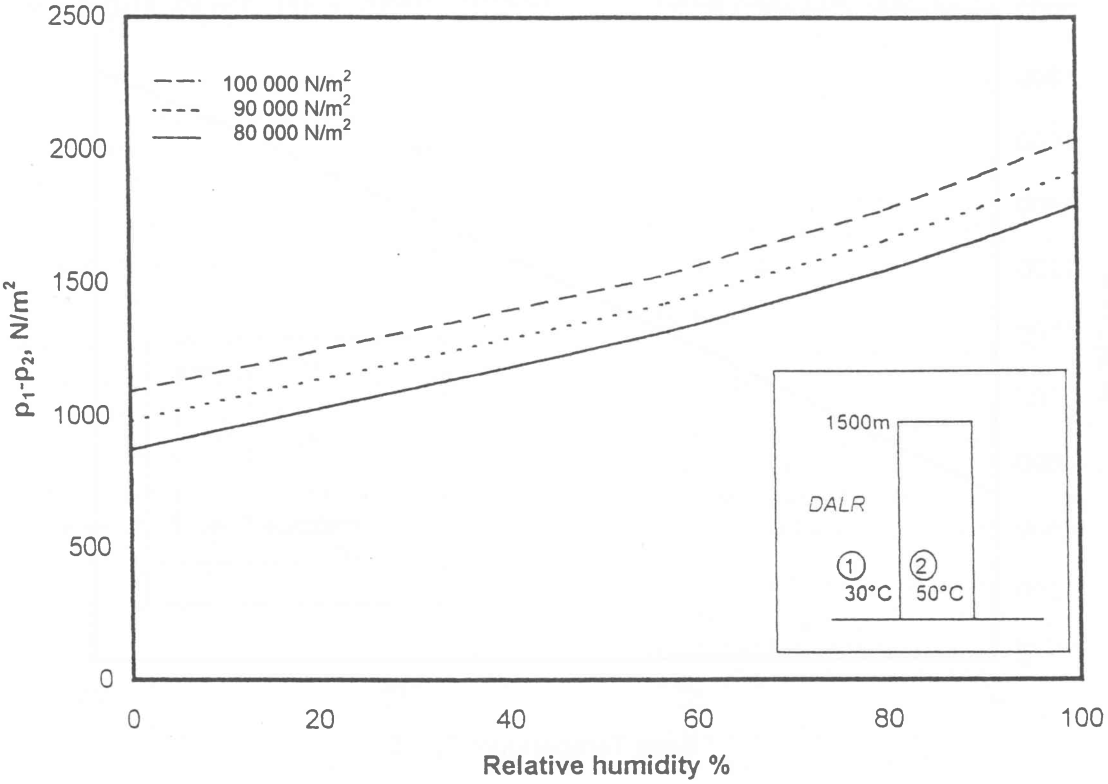

Figure 7 Influence of ambient pressure and humidity on potential

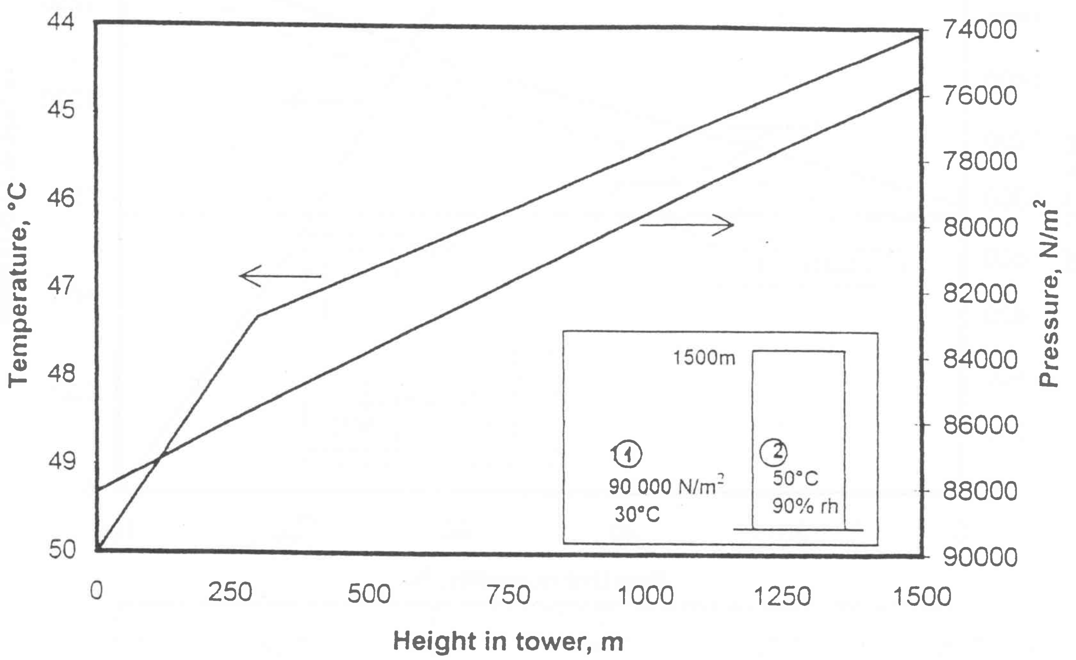

Figure 8 Temperature distribution in chimney

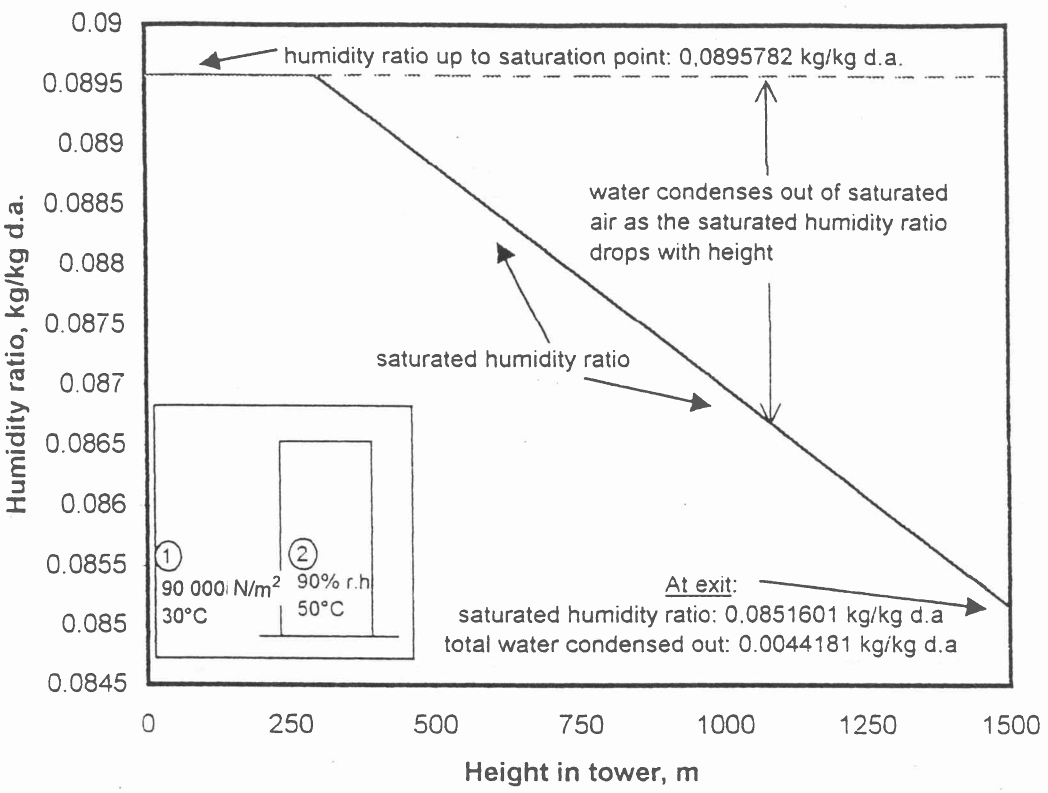

Figure 9 Humidity ratio at different elevations inside the chimney

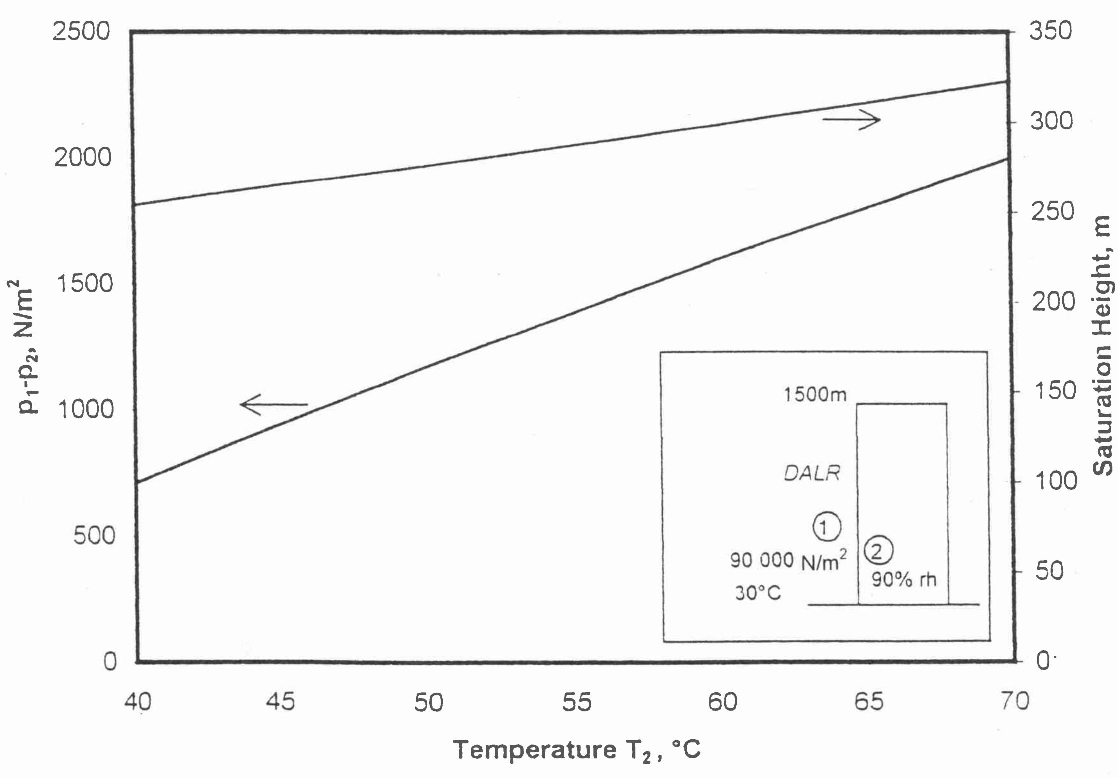

Figure 10 Influence of chimney inlet temperature on potential

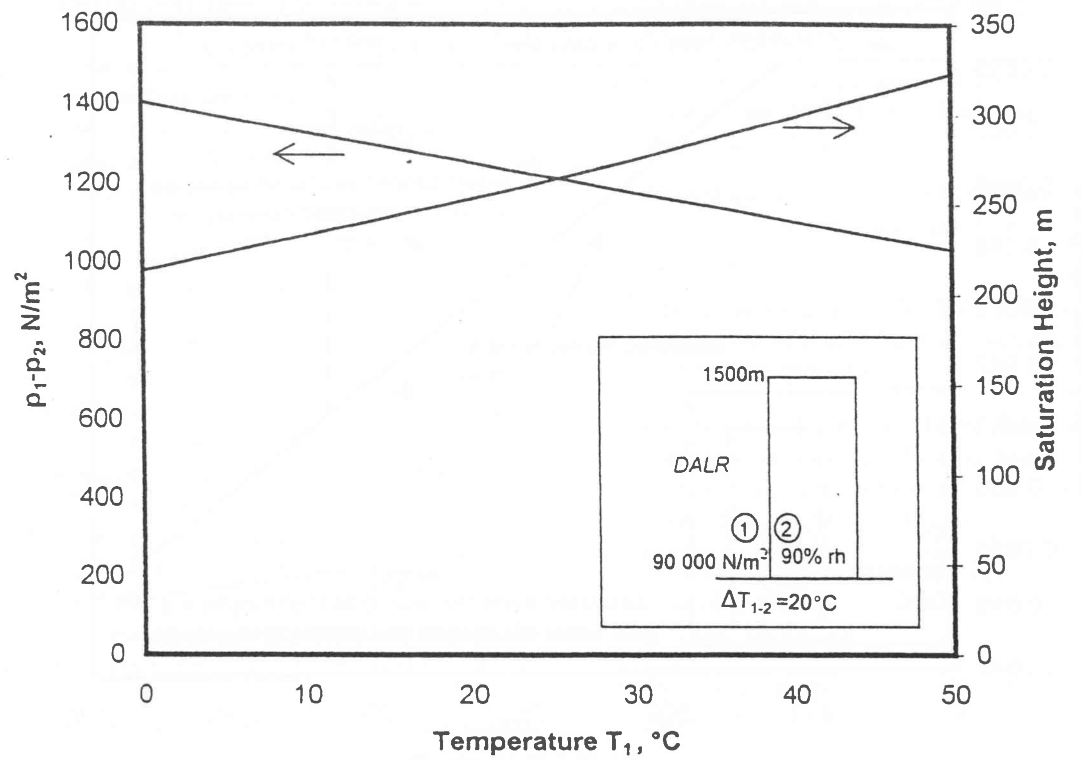

Figure 11 Influence of ambient temperature on potential

at the inlet to the chimney approaches 60%, the vapour near the top of the tower will become saturated. For higher values of inlet relative humidity, saturation will occur at lower elevations in the chimney, as shown in Figure 6. Above the elevation of saturation, equation (37) is applicable. Due to further cooling in this region, vapour will condense forming a cloud of water droplets.

Figure 7 shows the influence that different ambient pressures have on the potential. Typical temperature and pressure distributions in the chimney for a relative humidity of 90% at the chimney inlet are shown in Figure 8, whilst the humidity ratio at different elevations is shown in Figure 9. The influence of different chimney inlet temperatures on the potential and saturation height under these conditions is shown in Figure 10. Similarly the influence of different ambient temperatures is shown in Figure 11.

The influence of surface and upper layer inversions as well as the distribution and changes in the ambient relative humidity should also be considered in more refined models for determining potentials. Under certain operating conditions the formation of large droplets in the chimney may result in a significant reduction in effective potential.

References

1. Haaf M, Friedrich K, Mayr G & Schlaich J. Solar Chimneys Part 1: Principle and Construction of the Pilot Plant in Manzanares. *Int. J. Solar Energy*, **2**, 1983, pp.3–20.

2. Kröger DG. Air-cooled Heat Exchangers and Cooling Towers. Department of Mechanical Engineering, University of Stellenbosch, Stellenbosch, South Africa, 1998.