Compressible Flow Through Solar Power Plant Chimneys

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Chimneys as tall as 1500 m may be important components of proposed solar chimney power plants. The exit air density will then be appreciably lower than the inlet density. The paper presents a one-dimensional compressible flow approach for the calculation of all the thermodynamic variables as dependent on chimney height, wall friction, additional losses, internal drag and area change. The method gives reasonable answers even over a single 1500 m step length used for illustration, but better accuracy is possible with multiple steps. It is also applicable to the rest of the plant where heat transfer and shaft work may be present. It turns out that the pressure drop associated with the vertical acceleration of the air is about three times the pressure drop associated with wall friction. But flaring the chimney by 14 percent to keep the through-flow Mach number constant virtually eliminates the vertical acceleration pressure drop. [S0199-6231(00)03003-3]

Introduction

One of the major parts of a solar chimney power plant is the chimney itself (Fig. 1). When warmer than ambient air enters at the bottom of the chimney, the difference in density between it and that of the ambient air generates a pressure potential that causes the flow through the system and turbine. The pressure drops due to wall friction, loss coefficients and drag of obstructions in the chimney, and changes in static pressure due to changes in kinetic energy in contractions and diffuser shaped sections of the chimney affect the flow through and pressure drop over the turbine. The magnitude of the flow and the pressure drop over the turbine determine the maximum power that the turbine can extract from the flow. Haaf et al. [1], Haaf [2] and Von Backström and Gannon [3] and Gannon and Von Backström [4] have discussed solar chimney plant performance in more detail.

A method that can link the pressure drop in a chimney to its geometry and internal obstructions will be of great benefit in the analysis of solar chimney plants.

Objectives

The main objectives of this study are:

- To develop a method to calculate the variation in pressure for buoyant flow in a tall vertical chimney with wall friction, non-constant area and internal obstructions.
- To verify the method against simplified analytical solutions.
- To do a sensitivity analysis to determine the relative importance of the various parameters.
- To make recommendations regarding chimney layout.

Typical Chimney Construction

The chimney typically consists of the following:

- A chimney support section
- The turbine inlet
- The turbine
- The turbine diffuser
- A constant area duct or variable area diffusing section
- A protruding ring section that supports bracing wires, rods or beams
- Repetitions of the last two items
- A chimney exit

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Approach and Assumptions

The chimneys of solar chimney power plants may be as high as 1500 m. The maximum air velocity in the chimney will occur when the turbine blades are feathered and the flow resistance over the turbine is a minimum. Preliminary calculations show that the velocity will probably not exceed 50 m/s. The Mach numbers will then be below 0.15. Normally such flows would be considered to be incompressible, but in a tall chimney the change in density due to change in altitude also plays a role.

Assuming adiabatic conditions in the chimney, the temperature in the chimney will decrease at a rate of 9.76°C per 1000 m. That is about 15°C, or 5 percent in a 1500 m high chimney with an inlet temperature of 300 K. Under adiabatic conditions density ratios are proportional to temperature ratios to the power $1/(\gamma-1)$. The density then decreases by a factor of $0.95^{1/(\gamma-1)}=0.88$ for $\gamma=1.4$. Since a density decrease of 12 percent is not negligible, a compressible flow approach to the problem is appropriate.

The standard approach to compressible flow problems is to use the conservation equations for mass, momentum and energy to derive differential equations for the variation of each of the relevant variables in terms of the local Mach number. These differential equations have analytical solutions for some simple cases such as isentropic flow with variable area, frictionless flow with heat addition, or adiabatic flow with friction. For more complex cases however, where multiple factors simultaneously affect the flow, the solution requires numerical integration by Runge-Kutta or similar schemes.

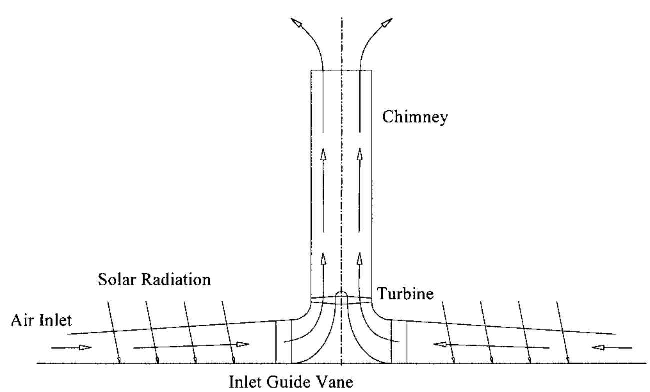

Fig. 1 Solar chimney schematic

In the solar chimney the following physical effects may influence the flow and specifically the change in stagnation or static pressure through the chimney:

- Wall friction
- Drag due to obstructions such as internal bracing or protrusions
- Flow separation at sudden contractions or expansions
- Cross sectional area change
- Heat transfer through the chimney walls
- Work extraction by the turbine
- The potential energy of the fluid

Preliminary calculations have shown that heat transfer through the chimney walls is negligible. The turbine deserves consideration, but separately. The effect of altitude variation on the potential energy of a gas is usually negligible, but for high chimneys it merits inclusion. For the sake of simplicity this first study will employ the one-dimensional approach, that is uniform property profiles across each section.

Formulation of the Problem

Application of the one-dimensional conservation equations to the control volume in Fig. 2 leads to:

Mass flow:

$$d\rho/\rho + dA/A + dV/V = 0 \tag{1}$$

Momentum:

$$dp + \rho V dV + \rho g dz + (4f dz/D)\rho V^2/2 + \delta F_D/A = 0 \tag{2}$$

Energy:

$$\delta W - \delta Q + dh + VdV + gdz = 0 \tag{3}$$

or:

$$-\delta W + \delta Q - gdz = dH \tag{4}$$

The normal definition of the stagnation enthalpy is:

$$H = h + V^2/2 \tag{5}$$

so that:

$$dH = dh + VdV \tag{6}$$

In problems where the potential energy term, gz is negligible, and work and heat transfer are zero, the energy equation merely states that the stagnation enthalpy is conserved. Such a convenient condition is possible even when the potential energy term is not negligible, but then the stagnation enthalpy definition should be:

$$H = h + V^2/2 + gz \tag{7}$$

with:

$$dH = dh + VdV + gdz. \tag{8}$$

The corresponding definition of stagnation temperature is then:

$$T = t + V^2/(2c_p) + gz/c_p \tag{9}$$

The additional gz/c_p term in the frequently used relationship between the static and stagnation temperature complicates the derivation of some of the other equations, however. A more convenient approach is to retain the standard definitions of stagnation enthalpy and stagnation temperature (without the gz term). The system of equations then requires only one modification, and that is that the stagnation enthalpy be reduced with the increase in potential energy over each altitude increase in Eq. (4). This is easy to do for one dimensional flow in a vertical chimney.

With these assumptions the normal set of equations for generalized steady one dimensional flow as derived by for example Zucrow and Hoffman [5] is applicable. This set of equations is solvable by numerically integrating the momentum equation, followed by appropriate substitution for other variables.

Solution Method

The differential equation for the effect of the relevant variables—area change, friction and drag, fluid weight, and stagnation temperature change—on Mach number variation in a vertical chimney is:

$$\begin{aligned} dM/M = &\{[1+M^2(\gamma-1)/2]/(1-M^2)\} \\ &\times\{-dA/A + [(f dz/D)\gamma M^2/2 + gdz/(Rt) + \delta F_D/(pA)] \\ &+ (dT/T)(1+\gamma M^2)/2\} \end{aligned} \tag{10}$$

The equation is derived from the vertical momentum equation divided by pA to make it dimensionless. Zucrow and Hoffman [5] initially included a term, $g\rho A dz/(pA) = gdz/Rt$ in the equation above, but discarded it during the derivation, as it is generally negligible. This term can be seen as an additional drag term equal to the weight of the fluid in the volume Adz.

Equation (10) must be integrated numerically over the height of the chimney. The starting values are the conditions at the bottom of the chimney, but there is no fundamental reason why a similar integration cannot be carried out right through, starting at the collector inlet and ending at the chimney exit. The effect of the turbine pressure drop must then be included as a drag force. The inlet values to be known are Mach number, area, diameter, pressure and stagnation temperature. During the integration process the value of friction factor must be calculated as function of Reynolds number and wall roughness. The lengths of the integration steps must be adjusted so that regions where obstructions, protrusions or additional losses occur fit neatly into one step. The drag force, δF_D is everywhere equal to zero except in such steps.

If the mass flow, static pressure, stagnation temperature and flow area are known at the starting point of the integration, find the Mach number there by solving the equation below:

$$m = A_1 p_1 M_1 \{[1+M_1^2(\gamma-1)/2]\gamma/(RT_1)\}^{0.5} \tag{11}$$

An easy way is to rewrite it as a quadratic equation for M_1^2:

$$M_1^2 = \{1/(\gamma-1)^2 + [m/(A_1 p_1)]^2 2RT_1/[\gamma(\gamma-1]\}^{0.5} - 1/(\gamma-1) \tag{12}$$

The static temperature follows from:

$$t_1 = T_1/[1+M_1^2(\gamma-1)/2] \tag{13}$$

Then find the stagnation enthalpy at the end of the integration step from the differential equation for energy conservation:

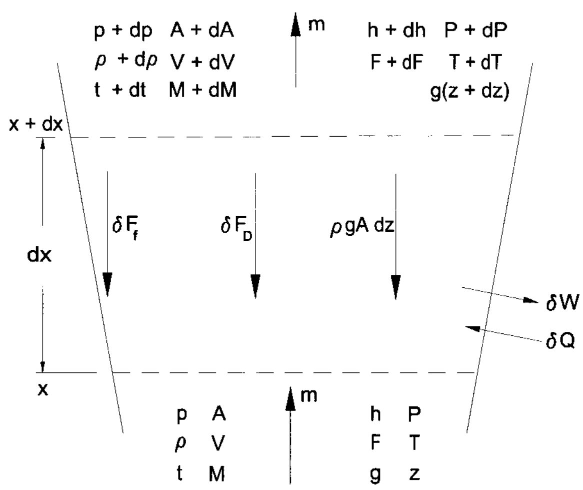

Fig. 2 Physical model for generalized steady one-dimensional flow

$$dH = -\delta W + \delta Q - g\,dz \tag{14}$$

Integration from the beginning of the integration step (position 1) to the end of the step (position 2) leads to:

$$H_2 - H_1 = -\Delta W + \Delta Q - g(z_2 - z_1) \tag{15}$$

where ΔW and ΔQ are work and heat inputs in a discrete integration step.

As pointed out before, $\Delta Q = 0$ for typical chimneys, and ΔW will have a non-zero value only over the turbine.

With A_2, M_2 and $T_2 = H_2/c_p$ known, the static pressure at the end of the integration step follows from the mass flow equation above:

$$p_2 = [m/(A_2 M_2)]/\{[1 + M_2^2(\gamma-1)/2]\gamma/(RT_2)\}^{0.5} \tag{16}$$

The stagnation pressure, P_2 at the end of the integration step follows from:

$$P_2 = p_2[1 + M_2^2(\gamma-1)/2]^{\gamma/(\gamma-1)} \tag{17}$$

Next calculate the static temperature as at the beginning of the integration step from the stagnation temperature and Mach number. The density is easy to calculate from the known pressure and temperature and so is the velocity from the density and flow area.

When the stagnation pressure at the chimney inlet is known, but not the static pressure, then the inlet static pressure and Mach number that will satisfy Eqs. (11) and (17) must be found by iteration.

The integration process is carried through to the chimney exit, where the static pressure must be equal to the ambient atmospheric static pressure at that altitude. If the calculated exit static pressure is higher (lower) than the ambient, the mass flow must be increased (reduced) iteratively until the pressures are equal.

The solution method described is equally applicable to the whole chimney plant including the collector, where of course $\Delta Q \neq 0$ and the flow area varies at a much greater rate than in the chimney. It is also applicable through the turbine as long as the turbine work extraction term remains in Eq. (4) and a turbine drag term represents the turbine pressure drop in Eqs. (2), (10) and following equations.

Simple Test Case: Zero Inlet Mach Number in An Ideal Chimney

We define an ideal chimney as one with no area change, internal obstructions, wall friction or additional losses, no heat transfer or work extraction. In addition, to test if the system of equations would reproduce the normal adiabatic lapse rate equations, we assume that the Mach approaches zero.

The differential equation for the chimney stagnation temperature drop follows from Eq. (14) for $\delta W = 0$ and $\delta Q = 0$:

$$dT = -g\,dz/c_p \tag{18}$$

Integrate from $z = 0$ where $T = T_1$ to a general height $z = z$ where $T = T$:

$$T = T_1 - (g/c_p)z \tag{19}$$

This is the well-known adiabatic temperature lapse rate equation, stating that the temperature decreases at 9.76°C per 1000 m altitude. To find the corresponding pressure lapse rate, first consider the Mach number ratio in a vertical chimney.

When M approaches zero, Eq. (10) approaches the following equation:

$$dM/M = \{1/1\{-dA/A + [0 + g\,dz/(Rt) + \delta F_D/(pA)] + (dT/T)1/2\} \tag{20}$$

For a constant area chimney $(dA = 0)$, with no internal drag $(\delta F_D = 0)$, it simplifies to:

$$dM/M = g\,dz/(Rt) + (dT/T)/2 \tag{21}$$

Substituting (18):

$$dM/M = g\,dz/(Rt) - g\,dz/(2c_pT) \tag{22}$$

Since $R = c_p(\gamma-1)/\gamma$, and for M approaching zero, t approaches T:

$$dM/M = [g\,dz/(c_pT)][\gamma/(\gamma-1) - 1/2] \tag{23}$$

$$= \{[g/(c_pT)]\{1/2(\gamma+1)/(\gamma-1)\}dz \tag{24}$$

To write T as a function of z, substitute (19):

$$dM/M = \{[1/2(\gamma+1)/(\gamma-1)][g/c_p]/[T_1 - (g/c_p)z]\}dz \tag{25}$$

Integrate to get an equation for the Mach number growth rate in a vertical chimney:

$$M/M_1 = [1 - (g/c_p)z/T_1]^{-1/2(\gamma+1)/(\gamma-1)} \tag{26}$$

By using Eq. (16) with the present assumptions, the chimney pressure ratio is:

$$p/p_1 = (M/M_1)^{-1}(T/T_1)^{0.5} \tag{27}$$

Substitute (26) and (19):

$$p/p_1 = [1 - (g/c_p)z/T_1]^{1/2(\gamma+1)/(\gamma-1)}[1 - (g/c_p)z/T_1]^{1/2} \tag{28}$$

Add the exponents:

$$p/p_1 = \{1 - gz/(c_pT_1)\}^{\gamma/(\gamma-1)} \tag{29}$$

This agrees with the equation for the pressure lapse rate corresponding to the adiabatic temperature lapse rate of $g/c_p = 9.81/1005 = 0.00976$°C/m.

It is gratifying to see that when the chimney inlet Mach number approaches zero, in the absence of friction, heat transfer, internal obstructions and work extraction, the system of equations to be solved reduces to the adiabatic lapse rate for temperature and pressure. Additionally, a useful equation for Mach number ratio in a vertical chimney presented itself in the derivation.

Sensitivity Analysis

Now that we have at least one exact solution to the system of equations we shall check how accurate a finite difference solution with one step over the 1500 m height is. We shall then use that as a benchmark to investigate the effect of a realistic inlet Mach number and other terms in the equation. It would be helpful to first consider the typical chimney geometry and operating conditions.

Numerical Values for Ideal 1500 m High 160 m Diameter Chimney. According to preliminary calculations a 200 MW peak power plant could have a solar collector diameter of 4 km and a chimney height of 1500 m and an inner diameter of 160 m. When sited at an elevation where the typical atmospheric conditions are 30°C (303,2K) and 90 kPa the air temperature rise could be 20°C in the collector (to 323,2 K), resulting in an optimal chimney through-flow velocity of 10 to 20 m/s. Assuming adiabatic lapse rates in the chimney, the temperature will drop by 14.6°C or 4.53 percent K through the chimney. The pressure ratio will then be $0.9547^{3.5} = 0.850$, the Mach number ratio $0.9547^{-3} = 1.1492$ and the density ratio $0.9547^{2.5} = 0.891$. The decrease in density ratio implies that in a constant area chimney, where ρV is constant, the velocity and dynamic pressure, $1/2\rho V^2$ will increase with the inverse ratio of the density to satisfy mass conservation. This implies that the dynamic pressure is 1/0.891 or 112.3 percent as high at the exit of a 1500 m high chimney as at the bottom. Since the chimney exit loss is equal to the dynamic pressure at its exit, the aerodynamic benefits of flaring the chimney to keep the Mach number constant, for example, are obvious.

Calculate the typical Mach number level at the bottom of a chimney from Eq. (12) and assuming a chimney diameter $D = 160$ m, and a flow area $A = 20110\ \mathrm{m}^2$, with mass flow m 386,000 kg/s and $p = 90$ kPa. Then:

$$M_1^2 = \{1/(\gamma-1)^2 + [m/(A_1 p_1)]^2 2RT_1/[\gamma(\gamma-1]\}^{0.5} - 1/(\gamma-1) \tag{12}$$

$$M_1^2 = \{1/0.4^2 + [386{,}000/(20{,}110\times 90{,}000)]^2 \times 2\times 287\times 323.2/[1.4(0.4]\}^{0.5} - 1/0.4$$
$$M_1 = 0.0549 \tag{30}$$

At the bottom of the chimney the static temperature is then, $t_1 = 323.0$ K, the sonic velocity, $a_1 = 360.3$ m/s and the velocity, $V_1 = 19.8$ m/s. The density, $\rho_1 = 0.971\ \mathrm{kg/m^3}$ and the dynamic pressure, $1/2\rho V^2 = 190$ Pa.

Discretization Error Investigation. Substitute $T - T_1 = \Delta T$ in equation (19) and divide by T_1:

$$\Delta T/T_1 = -gz/(c_p T_1) \tag{31}$$

Note that $gz/(c_p T_1)$ is the ratio of the change in potential energy over the chimney height to the inlet enthalpy per unit mass of the chimney flow. We shall use the symbol E_1 for this ratio.

$$\Delta T/T_1 = -E_1 \tag{32}$$

Write the two-term binomial expansion of (26):

$$M/M_1 = (M_1 + \Delta M)/M_1 \approx [1 + 1/2(\gamma+1)/(\gamma-1) g\Delta z/(c_p T_1)] \tag{33}$$

Thus, for $\gamma = 1.4$:

$$\Delta M/M_1 \approx 1/2(\gamma+1)/(\gamma-1)E_1 = 3.0E_1 \tag{34}$$

Similarly, from (29):

$$1 + \Delta p/p_1 \approx 1 - [\gamma/(\gamma-1)]E_1 \tag{35}$$

For $\gamma = 1.4$:

$$\Delta p/p_1 \approx -3.5E_1 \tag{36}$$

By logarithmic differentiation of the equation of state, remembering that $t \Rightarrow T$ when $M \Rightarrow 0$:

$$\Delta\rho/\rho = \Delta p/p - \Delta T/T \tag{37}$$

So, from (36) and (32):

$$\Delta\rho/\rho_1 \approx -3.5E_1 - (-E_1) = -2.5E_1 \tag{38}$$

Table 1 compares results from the single step discretized solution to exact results for a 1500 m chimney, with $g = 9.81\ \mathrm{m/s^2}$, $c_p = 1005$ J/(kg K) and $T_1 = 323.2$ K. From Eq. (31) calculate $\Delta T/T_1 = E_1 = -g\Delta z/(c_p T_1) = -0.0453$. Subscripts 1 and 2 indicate the bottom and top of the chimney. The errors are based on magnitudes, disregarding signs.

For a 1500 m step length the discretization errors are less than 6 percent, but can be reduced more or less proportionally to the integration step length. For 15 m step lengths they should be less than 0.06 percent. Since the discretized values are reasonably accurate even for tall chimneys, they will serve as a basis for further initial investigations.

The dimensional differences are in Table 2.

As rules of thumb for low Mach number flow in an ideal chimney we may assume the following: the temperature decreases at the rate of 10°C per 1000 m, the pressure decreases at 10 kPa per 1000 m, the Mach number increases at 10 percent per 1000 m and the dynamic pressure at 10 percent per 1000 m.

Table 1 Comparison between exact and discretized changes in $\Delta T/T_1$, $\Delta M/M_1$, $\Delta p/p_1$ and, $\Delta\rho/\rho_1$ for 1500 m high chimney when $M_1 \Rightarrow 0$

Normalized difference	Exact (%)	Discre-tized	% Error
Temperature difference, $(T_2 - T_1)/T_1$	-4.53	-4.53	0.0
Mach number difference, $(M_2-M_1)/M_1$	14.22	13.59	4.4
Pressure difference, $(p_2 - p_1)/p_1$	-14.98	-15.86	5.9
Density difference, $(\rho_2 - \rho_1)/\rho_1$	-10.94	-11.33	3.5

Table 2 Comparison between exact and discretized changes in ΔT, ΔM, Δp and, $\Delta\rho$ for 1500 m high chimney when $M_1 \Rightarrow 0$

Dimensional difference	Exact	Discre-tized
Temperature difference, $(T_2 - T_1)$ [K]	-14.64	-14.64
Pressure difference, $(p_2 - p_1)$ [Pa]	-13480	-14270
Density difference, $(\rho_2 - \rho_1)$ [kg/m^3]	-0.1062	-0.1010

Effect of Inlet Mach Number on Mach Number Change in a 1500 m High Chimney. We still assume an ideal chimney but we now drop the assumption that M_1 approaches zero. It is evident from (18) that Mach number has no effect on stagnation temperature, since z alone determines it. Equation (10) gives the Mach number change:

$$[dM/M]/\{[1 + M^2(\gamma-1)/2]/(1-M^2)\} = [gdz/(Rt) + (dT/T)(1+\gamma M^2)/2] \tag{39}$$

Substitute $t = T/[1 + M^2(\gamma-1)/2]$ and $R = c_p(\gamma-1)/\gamma$:

$$[dM/M]/\{[1 + M^2(\gamma-1)/2]/(1-M^2)\} = \{[gdz/(c_p T)][\gamma/(\gamma-1)][1 + M^2(\gamma-1)/2]^2 + (dT/T)[(1+\gamma M^2)/2]\} \tag{40}$$

Eliminate $gdz/(c_p T)$ through (18):

$$[dM/M]/\{[1 + M^2(\gamma-1)/2]/(1-M^2)\} = \{(dT/T)[-\gamma/(\gamma-1)][1 + M_1^2(\gamma-1)/2] + (dT/T)(1+\gamma M_1^2)/2\} \tag{41}$$

$$= [-\gamma/(\gamma-1) - M^2\gamma/2 + 1/2 + M^2\gamma/2](dT/T) \tag{42}$$

$$= [-1/2(\gamma+1)/(\gamma-1)](dT/T) \tag{43}$$

Discretize:

$$\Delta M/M_1 = [-1/2(\gamma+1)/(\gamma-1)] \times [1 + M_1^2(\gamma-1)/2]/(1-M_1^2)](\Delta T/T_1) = C_{MT}(\Delta T/T_1) \tag{44}$$

Where:

Table 3 Effect of chimney inlet Mach number, M_1 on fractional Mach number change $\Delta M/M_1$ for ideal 1500 m high chimney

M_1 = chimney inlet Mach number	C_{MT} = Ratio of change in M to change in T	% $\Delta M/M_1$ change compared to when M_1=0
0.00	-3.0000	0.00
0.05	-3.0090	0.30
0.10	-3.0364	1.21
0.20	-3.1500	5.00
0.40	-3.6857	22.86

$$C_{MT}=(\Delta M/M_1)/(\Delta T/T_1)$$
$$=[-1/2(\gamma+1)/(\gamma-1)][1+M_1^2(\gamma-1)/2]/(1-M_1^2)] \tag{45}$$

The fractional change in $\Delta M/M_1$ compared to $\Delta M/M_1$ when M_1 approaches zero, is given by:

$$[(\Delta M/M_1)-\Delta M/M_1)_{M1\Rightarrow 0}]/[(\Delta M/M_1)_{M1\Rightarrow 0}]$$
$$=(|C_{MT}|-|-3.0|)/|-3.0| \tag{46}$$

It is expressed as a percentage in Table 3.

As long as the chimney inlet Mach number, M_1 is below 0.05 it has a small effect on its own rate of change. But as the inlet Mach number increases, its rate of change increases quadratically. For $M_1=0.05$ the Mach number rate of change is 0.30 percent larger than when M_1 approaches zero. We shall see that the effect of this small acceleration on the pressure drop in the chimney is important even at that low Mach number.

Effect of Inlet Mach Number on Pressure Change in an Ideal 1500 m High Chimney. We shall now investigate how the inlet Mach number level affects the pressure drop in an ideal chimney. To evaluate that, we have to determine how dp/p varies when M_1 is not zero. Although the flow area is constant in an ideal chimney we treat A as a variable for later use. Start with Eq. (16), written for a general position (chimney height):

$$p=[m/(AM)]/\{[1+M^2(\gamma-1)/2]\gamma/(RT)\}^{0.5} \tag{47}$$

Apply logarithmic differentiation:

$$dp/p=-dA/A-dM/M$$
$$-\{[M^2(\gamma-1)/2]/[1+M^2(\gamma-1)/2]\}dM/M$$
$$+(1/2)dT/T \tag{48}$$

Simplify:

$$dp/p=-dA/A$$
$$-\{[1+M^2(\gamma-1)]/[1+M^2(\gamma-1)/2]\}dM/M$$
$$+(1/2)dT/T \tag{49}$$

Define:

$$C_{pM}=-[1+M_1^2(\gamma-1)]/[1+M_1^2(\gamma-1)/2].$$

Then:

$$\Delta p/p_1=-dA/A+C_{pM}\Delta M/M_1+C_{pT1}\Delta T/T_1 \tag{50}$$

where:

Table 4 Effect of chimney inlet Mach number, M_1 on pressure change, $\Delta p/p_1$ and acceleration pressure drop in ideal 1500 m high chimney

M_1	C_{pM} =Ratio of change in p to change in M	C_{pT} = Ratio of change in p to change in T	$\Delta p/p_1$ change (%)	Acceleration pressure drop [Pa]
0.00	-1.000	3.5000	0.00	0.0
0.05	-1.0005	3.5105	0.30	43
0.10	-1.0020	3.5424	1.21	173
0.20	-1.0079	3.6750	5.00	714
0.40	-1.0310	4.3000	22.86	3262

$$C_{pT1}=(\Delta p/p_1)/(\Delta T/T_1)=0.5 \tag{51}$$

Substitute (44):

$$\Delta p/p_1=-dA/A+C_{pM}C_{MT}(\Delta T/T_1)+C_{pT1}\Delta T/T_1 \tag{52}$$

$$\Delta p/p_1=-dA/A+(C_{pM}C_{MT}+C_{pT1})\Delta T/T_1=C_{pT}\Delta T/T \tag{53}$$

The fractional change in $\Delta p/p_1$ compared to $\Delta p/p_1$ when M_1 approaches zero, is given by:

$$[(\Delta p/p_1)-\Delta p/p_1)_{M1\Rightarrow 0}]/[(\Delta p/p_1)_{M1\Rightarrow 0}]=[|C_{pT}|-3.5)]/3.5 \tag{54}$$

It is expressed as a percentage in Table 4, and in the last column as the pressure drop caused by vertical acceleration in the absence of frictional and other loss mechanisms.

It is clear from Table 4 that the inlet Mach number at the bottom of the chimney should be extremely low to avoid a large increase in the chimney pressure drop compared to the no-flow condition. The additional pressure drop increases approximately with the square of the inlet Mach number. Even at a low Mach number of 0.05 the additional pressure drop due to acceleration (using the discretized fractional stationary pressure drop of -0.1586) is $0.0030\times(-0.1586)\times 90{,}000=43$ Pa. Note that this is not a frictional pressure drop but is a pressure drop due to the weight of the fluid when in vertical motion. Due to the density drop through the chimney the flow must accelerate through it. The acceleration requires an additional pressure gradient above the one for stationary conditions in the chimney. We shall now investigate the effect of fluid friction, drag and loss coefficients on the pressure drop.

Effect of Fluid Friction, Drag and Loss on Mach Number in a 1500 m High Chimney. The drag term, $\delta F_D/(pA)$ can be seen as including any so-called minor or additional losses such as those caused by sudden expansions for example. Such a pressure loss is usually written as a coefficient, K multiplied by the dynamic pressure, that is:

$$\delta p=K\rho V^2/2 \tag{55}$$

so that:

$$\delta F_k=KA\rho V^2/2 \tag{56}$$

and:

$$\delta F_k/(pA)=K\gamma M^2/2 \tag{57}$$

Table 5 Effect of chimney inlet Mach number, M_1 on Mach number change, $\Delta M/M_1$ for friction coefficient=0.0084, (FD_1 =0.07901) in 1500 m high chimney

M_1	% Change in Mach number due to T	C_{MFD} = Ratio of change in M to change in FD_1	% Change in Mach number due to FD_1	% $\Delta M/M_1$ change due to friction	% $\Delta M/M_1$ change due to acceler-ation
0.00	13.591	0.00000	0.000	0.00	0.00
0.05	13.632	0.00176	0.014	0.10	0.30
0.10	13.756	0.00708	0.056	0.41	1.21
0.20	14.270	0.02940	0.232	1.63	5.00
0.40	16.697	0.13760	1.087	6.51	22.86

When written in this form any drag or pressure loss term can be included in the skin friction term and treated similarly. For a constant area chimney, Eq. (10) then becomes:

$$dM/M=\{[1+M^2(\gamma-1)/2]/(1-M^2)\}[(fdz/D)\gamma M^2/2 + gdz/(Rt)+K\gamma M^2/2+(dT/T)(1+\gamma M^2)/2] \quad (58)$$

Collect terms and write t in terms of T and M, as in the derivation of (40):

$$dM/M/\{[1+M^2(\gamma-1)/2]/(1-M^2)\} = [(fdz/D)+K](\gamma M^2/2)] + [gdz/(Rt)+(dT/T)(1+\gamma M^2)/2] \quad (59)$$

Treat the second term on the right in (59) as in the derivation of (45). Then:

$$\Delta M/M_1=[(f_1 z/D_1)+K]C_{MFD}+C_{MT}\Delta T/T_1 \quad (61)$$

where:

$$C_{MFD}=(\gamma M_1^2/2)[1+M_1^2(\gamma-1)/2]/(1-M_1^2) \quad (60)$$

Then:

$$\Delta M/M_1=C_{MFD}FD_1+C_{MT}\Delta T/T_1 \quad (62)$$

where FD_1 is the friction-drag-loss term.

At chimney inlet conditions the density is 0.9705 kg/m^3, the dynamic viscosity is 1.95×10^{-5} Ns/m^2 and the velocity is 19.8 m/s. For a chimney diameter of 160 m the Reynolds number is then 1.57×10^8. The relative roughness for concrete is 2 mm. The Haaland equation [7] cited by White [6], then gives $f_1 = 0.008428$. Then $f_1 z/D = 0.07901$. As a first case neglect drag, protrusions and other loss coefficients, and take the additional loss coefficient, $K=0.0$.

It is clear from Table 5 that the additional effect of the assumed friction coefficient on Mach number change is limited to a fraction (about one third) of the effect of vertical acceleration at the typical operating inlet Mach number of about 0.05. Each additional increase in loss coefficient, K of 0.079 will have about the same small effect.

Table 6 Effect of chimney inlet Mach number, M_1 on Mach number change, $\Delta M/M_1$ for $\Delta A/A_1$=0.013591 in a 1500 m high chimney

M_1	% Change in Mach number due to T	C_{MA}= ratio of change in M to change in A	% Change in Mach number due to A	Total $\Delta M/M_1$ change (%)
0.00	13.591	-1.0000	-13.591	0.00
0.05	13.632	-1.0030	-13.632	0.00
0.10	13.756	-1.0121	-13.756	0.00
0.20	14.270	-1.0500	-14.270	0.00
0.40	16.697	-1.2286	-16.697	0.00

We have previously seen that the percentage change in pressure is the same as that in Mach number (except for a sign change). The conclusion is that in practice the effect of chimney skin friction on chimney pressure loss will be small.

Effect of Area Change on Mach Number in a 1500 m High Chimney. The most effective way of controlling the Mach number distribution through the chimney is to vary the area ratio, A. For a given chimney height an increase in area ratio is the only way of reducing the Mach number or its rate of increase through the chimney. For a variable area chimney with no friction or additional losses, a derivation analogous to the previous one leads to the following relationship:

$$dM/M=-dA/A[1+M^2(\gamma-1)/2]/(1-M^2)+C_{MT}\Delta T/T_1 \quad (63)$$

$$\Delta M/M_1=C_{MA}\Delta A/A_1+C_{MT}\Delta T/T_1 \quad (64)$$

It is evident from Table 6 that the increase in Mach number in the chimney can be entirely eliminated by a relatively small (14 percent) increase in chimney area and that the required area change is independent on Mach number. Greater changes in chimney flow area will lead to reductions in Mach number and increases in static pressure in the chimney.

Performance of a Typical Chimney

The condition that determines flow through a chimney is that the exit pressure at the top of the chimney must be equal to the ambient atmospheric pressure at that altitude. Since the temperature of the air in the chimney is higher and the density lower than outside, the noflow difference in static pressure in the chimney (pressure at bottom minus pressure at top) will be lower than outside. This difference will cause the flow to accelerate until the chimney exit pressure is equal to the ambient. As the flow in the chimney accelerates and the Mach number increases, additional pressure drop mechanisms will develop as discussed in the text. We shall now look a three typical through-flow velocities of 10 and 20 m/s. The initial air standard analysis (Von Backström, [3]) indicated that 19.8 m/s would be close to the expected velocity, but a later more detailed analysis that included the solar collector (Gannon and von Backström, [4]) indicated 10 m/s as closer to the optimum for maximum power extraction.

The equivalent sand roughness of the concrete chimney walls is 2 mm. The typically expected chimney inlet loss coefficient is

Table 7 Effect of chimney inlet velocity, V_1 on various pressures and pressure drops in a 1500 m high, 160 m diameter chimney

Inlet velocity ⇒	$V_1 = 10$ m/s	$V_1 = 20$ m/s	% of inlet dynamic
Reference pressures	[Pa]	[Pa]	
Inlet stagnation pressure	90000	90000	-
Inlet dynamic pressure	48	194	100
Inlet static pressure	89952	90194	-
Exit dynamic pressure	55	218	112
No-flow Δp	14270	14270	-
Pressure loss mechanism ⇓			
Vertical acceleration Δp	-13	-53	27
Friction Δp	-4	-15	8
Chimney inlet Δp, K = 0.25	-12	-49	25
Chimney internal Δp, K = 0.25	-12	-49	25
Sum of pressure losses	-41	-166	85
Sum of losses + exit dynamic	-96	-384	-198
Ideal diffuser pressure rise, $\Delta A/A_1 = 0.13591$	+13	+53	27

about 0.25 and we assumed the same value for the internal chimney bracing by cables or rods. Design and experimentation must still finalize them.

The following is evident from Table 7:

- The major pressure drop is the no-flow pressure drop of 14,270 Pa
- The exit dynamic pressure is 12 percent larger than the inlet dynamic pressure
- The vertical acceleration pressure drop is 27 percent of the inlet dynamic pressure
- The friction pressure drop is about one third of the vertical acceleration pressure drop
- The vertical acceleration pressure drop may be larger than the chimney inlet pressure drop
- The total expected chimney pressure drop is of the order of twice the chimney inlet dynamic pressure for a constant area chimney
- The vertical acceleration pressure drop can be eliminated by flaring the chimney by about 14 percent

Conclusions

The paper presents a one-dimensional compressible flow approach for the calculation of all the thermodynamic variables as dependent on chimney height, wall friction, additional losses, internal drag and area change. For the no-flow condition, the differential equations reduce to the adiabatic temperature lapse rate and associated pressure equation. The discretized method predicted no-flow pressure and density differences to within 6 percent and 4 percent even over a single 1500 m step length, used for illustration. Good accuracy is possible with multiple (say 100) steps. The method is also applicable to the complete plant where heat transfer and shaft work may be present.

It turns out that the pressure drop associated with the vertical acceleration of the air is about three times the pressure drop associated with wall friction. So reducing the wall friction by smoothing the walls will have a smaller than usual effect. But flaring the chimney by 14 percent to keep the through-flow Mach number constant, virtually eliminates the vertical acceleration pressure drop. Since the flow is not decelerating, excellent pressure recovery can be expected. However, flow stratification resulting from hot air from near the collector floor rushing up in the middle of the chimney may inhibit the pressure recovery.

Since the vertical acceleration pressure drop is so large, no effort should be spared to design the chimney in such a way that the flow area increases with height, even if only marginally. A 14 percent increase in area is equivalent to a 13 m increase in diameter for a 160 m diameter chimney. Here follows a list of possible ways of increasing the flow area with height:

- The reduction in chimney wall width with altitude can be on the inside rather than on the outside
- Where protrusions occur to anchor the cross bracing, the higher chimney section can have a slightly larger diameter than the lower section
- Buttresses in the lower sections of the chimney should preferably be on the inside, where they reduce the flow area, rather than on the outside
- Since wall friction has such a small effect, the lower sections of the chimney where the walls are thicker may be fluted (folded zig-zag) inwards to reduce the flow area.

A relatively short diffuser on the chimney exit is likely to be less effective than preventing the flow acceleration in the first place.

Nomenclature

A = flow area [m^2]
C = coefficient
c_p = specific heat [J/kg K]
D = chimney inside diameter [m]
E = potential to inlet energy ratio
F_D = drag force [N]
f = friction coefficient
g = gravitational acceleration [m/s^2]
H = stagnation enthalpy [J/kg]
h = static enthalpy [J/kg]
K = pressure drop coefficient
M = Mach number
m = mass flow [kg/s]
P = stagnation pressure [Pa]
p = static pressure [Pa]
Q = heat per unit mass [J/kg]
R = gas constant [J/kg K]
T = stagnation temperature [K]
t = static temperature [K]
V = velocity [m/s]
W = work per unit mass [J/kg]
z = chimney height [m]

Greek

γ = specific heat ratio
ρ = density [kg/m^3]

Prefix

Δ = change in value
δ = prefix: elemental value

Subscript

D = drag
MA = effect of A on M
MFD = effect of friction etc. on M
MT = effect of T on M
pM = effect of M on p
pT = (also $pT1$) effect of T on p
1 = inlet (or coefficient number)
2 = exit (or coefficient number)

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