THE SUITABILITY OF THE INFINITE NTU MODEL FOR LONG-TERM ROCK BED THERMAL STORAGE PERFORMANCE SIMULATION

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ABSTRACT

Packed beds incorporating rock as the storage medium and air as the heat transfer fluid have been proposed as a cost-effective approach for thermal storage in solar power plants. In order to assess the viability of rock bed thermal energy storage (TES), it is essentially necessary to predict the air and rock temperature profiles through the bed during charging and discharging. A number of models are available for this purpose, including, at the more basic level, the popular Hughes "E-NTU" model and the simplified "Infinite NTU" model. Unlike the E-NTU method, the Infinite NTU method assumes an infinite heat transfer coefficient between the air and the rock, which implies equivalent air and rock temperature profiles. Typically, the more detailed a model, the greater the computational effort required to solve it. For long-term analysis, a time-efficient model is necessary to prevent excessively long computation times. This paper evaluates the comparative accuracy and computational efficiency of the E-NTU and Infinite NTU models when simulating the short- and long-term performance of CSP rock bed TES systems. On the basis of these comparisons, the appropriateness of employing the less realistic but less costly Infinite NTU model in the long-term simulation of CSP rock bed TES systems is evaluated.

INTRODUCTION

Thermal energy storage (TES) is a key component of concentrating solar power (CSP) plants. Although molten salt TES has established itself as the commercial state of the art [1], it is relatively expensive and therefore lower cost systems are desirable. Packed beds incorporating rock as the storage medium and air as the heat transfer fluid have been proposed [2] as a more cost-effective approach, due to the low cost of rock and abundant availability of air.

NOMENCLATURE

Roman Symbols

A	[m^2]	Area
c	[J/kg-K]	Specific heat capacity
D	[m]	Equivalent particle diameter
G	[kg/s-m^2]	Mass flux
h_v	[W/m^3-K]	Volumetric heat transfer coefficient
k	[W/m-K]	Thermal conductivity
L	[m]	Bed length
$\dot{m}$	[kg/s]	Mass flow rate
NTU	[-]	Number of transfer units
t	[s]	Time
T	[K or °C]	Temperature
z	[m]	Axial coordinate in the flow direction
Δz	[m]	Bed segment length

Greek Symbols

ε	[-]	Bed porosity
ρ	[kg/m^3]	Density
τ	[-]	Thermal time constant

Subscripts

cs	Cross-section
$charge$	Charge mode
$discharge$	Discharge mode
f	Fluid phase, air
in	Bed inlet
m	Node number
p	Isobaric
s	Solid phase, rock

In order to assess the viability of rock bed TES systems, it is necessary to study their long term charge-discharge thermal performance and the associated effect on power generation. Essentially, this requires the ability to predict the air and rock temperature profiles through the bed during charging and discharging. A number of models are available for this purpose, including, at the more basic level, the popular Hughes "E-

NTU" (Effectiveness – Number of Transfer Units) model and the simplified "Infinite NTU" model [3,4,5]. Unlike the E-NTU method, the Infinite NTU method assumes an infinite heat transfer coefficient between the air and the rock, which implies equivalent air and rock temperature profiles. There are more detailed models such as the "Continuous Solid Phase" model whose governing equations account for thermal conduction through the rock and air [6]. Typically, the more detailed a model, the greater the computational effort required to solve it. This is infrequently problematic for the simulation of a few charge-discharge cycles, but for long-term analysis in the context of annual CSP plant performance simulation, a time-efficient model is necessary to prevent excessively long computation times. This is especially the case when conducting plant-level conceptual design or optimisation studies.

This paper evaluates the comparative accuracy and computational efficiency of the E-NTU and Infinite NTU models when simulating the short- and long-term performance of CSP rock bed TES systems. For short-term operation over charge and discharge cycles, comparisons are drawn between model predictions and data derived from an experimental rock bed with peak temperatures near 300 °C. For long-term operation, performance predictions generated by either model for many charge-discharge cycles are compared over various time intervals. On the basis of these comparisons, the appropriateness of employing the less realistic but less costly Infinite NTU model in the long-term simulation of CSP rock bed TES systems is evaluated.

THEORY
The Schumann Equations

The assumptions underlying the E-NTU model [3] are those of the Schumann model [7]. Flow is assumed to be one-dimensional with a uniform axial flow speed, there is no thermal loss from the bed walls, the internal thermal resistance of each solid particle is neglected, and radiant and conductive heat transfer through the bed is assumed to be negligible. For air-rock beds, the thermal capacitance term of the air can be neglected. These assumptions give rise to the Schumann equations. For the fluid phase, in this case air,

$$\frac{\partial T_f}{\partial z} = \frac{h_v A_{cs}}{\dot{m} c_{p,f}}(T_s - T_f) = \frac{NTU}{L}(T_s - T_f) \tag{1}$$

where

$$NTU = h_v A_{cs} L /(\dot{m} c_{p,f}) = h_v L /(G_f c_{p,f}) \tag{2}$$

For the solid phase, in this case rock,

$$\frac{\partial T_s}{\partial t} = \frac{h_v}{(1-\varepsilon)\rho_s c_s}(T_f - T_s) = \frac{NTU}{\tau}(T_f - T_s) \tag{3}$$

with the thermal time constant defined as

$$\tau = \rho_s(1-\varepsilon)A_{cs}Lc_s/(\dot{m} c_{p,f}) \tag{4}$$

The E-NTU Model

The Hughes E-NTU method relies on a bed segment temperature approximation to expedite the numerical solution of discretized forms of Eqs. (1) and (3). The rock temperature is assumed to be constant in a given segment, while the air temperature has an exponential profile [3]. The air temperature at the next segment ($m+1$) is expressed as:

$$T_{f,m+1} = T_{f,m} - (T_{f,m} - T_{s,m})(1 - e^{-NTU(\Delta z/L)}) \tag{5}$$

The governing equation for the rock temperature can be expressed in terms of Eq. (5) as

$$\frac{dT_s}{dt} = \frac{L}{\Delta z}\frac{1}{\tau}(T_{f,m} - T_{s,m})(1 - e^{-NTU(\Delta z/L)}) \tag{6}$$

One of a number of numerical integration strategies may be used to solve Eq. (6) and thus Eq. (5). Duffie and Beckman make use of Euler-stepping [5].

The Infinite NTU Model

If one makes the assumption that the fluid and solid phases are in thermal equilibrium throughout the packed bed, the model reduces to a single governing equation, Eq. (7), termed the Infinite NTU model by Hughes et al. (1976).

$$\frac{dT_b}{dt} = -\frac{\dot{m} c_{p,f}}{(1-\varepsilon)A_{cs}\rho_s c_s}\frac{\partial T}{\partial z} \tag{7}$$

Discretisation and numerical integration of this equation is simple and demands significantly lower computational effort in comparison to the E-NTU model. However, the Infinite NTU approximation relies on the assumption that the convective heat transfer between the phases is sufficiently high that it results in local thermal equilibrium. In this regard, Duffie and Beckman state that values of NTU higher than 25 give long term performance predictions essentially the same as those obtained by assuming a finite NTU [5]. They also remark that even for NTU values as low as 10, the infinite NTU method should still provide reasonable long term performance predictions.

EXPERIMENTAL DETAILS

Charge and discharge tests were conducted on a packed bed of rock 1.5 m in length and nominally 1 m² in cross-sectional area. A diagram of the test rig is shown in Figure 1. A diesel burner provides heat to the airstream to charge the system at temperatures up to 600 °C. The flow control valves allow the flow direction to be reversed for discharging of the bed. The air temperature is measured by type K thermocouples, positioned in layers as shown in Figure 2. Each layer consisted of six thermocouples, and the average of each layer was used. The air flow was measured by means of a bellmouth upstream of the fan, and an orifice plate in the exhaust duct. The stainless steel

test section was insulated internally and externally. Further detail is given in [8].
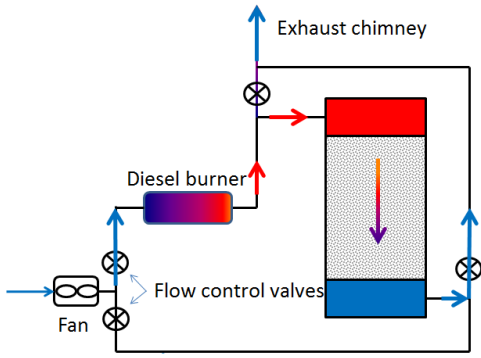

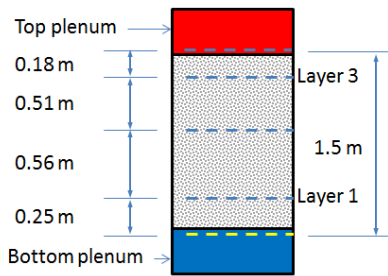**Figure 1: Test rig layout and flow direction during charging (from [8])**

Figure 2: Thermocouple positioning (from [8])

The test section was filled with dolerite, an igneous rock. The bed characteristics and rock properties are listed in Table 1.

Table 1: Bed characteristics and rock properties

Parameter	Value
L	1.489 m
A_{cs}	0.9781 m^2
D	0.039 m
ε	0.45
ρ_s	2869 kg/m^3
c_s (50-60 °C)	815 J/kgK

For simulation purposes, the specific heat capacity of the rock, c_s, was evaluated as a function of local rock temperature according to a general correlation [9].

In the numerical solution of the E-NTU and Infinite NTU equations, grid independence in both space and time should be demonstrated in order for discretisation error to be minimised. As such, spatial grid resolution sensitivity studies were conducted for both models with respect to analytical solutions to the Schumann and Infinite NTU equations, with time step size suitably adjusted to yield step size-independent solutions.

For the Schumann equations, the analytical solution provided in [11] was used, whereas for the Infinite NTU equation the Heaviside-type analytical solution was obtained by means of Laplace transform techniques. For this study, bed characteristics and rock properties typical of a large CSP rock bed were applied.

SIMULATION DETAILS

All E-NTU and Infinite NTU modelling was undertaken using the computational software EES [10]. For integration in time, EES's semi-implicit predictor-corrector integral function was used which enabled the application of significantly larger time step sizes than permitted by purely explicit schemes. All air properties were evaluated at the local bed air temperature by means of EES' built-in ideal gas property functions.

For the experimental study, when simulating the charge and discharge operation of the test bed, simulations were driven by air temperature measurements made at the bed inlet for either case, in addition to the associated air mass flow rate data. These data were provided at 10 second intervals.

The long-duration numerical study considered the performance of a packed bed with characteristics appropriate to operation in a large open volumetric receiver CSP plant, as reflected in Table 2.

Table 2: Bed characteristics and rock properties associated with the long-duration numerical study.

Parameter	Value
L	15 m
A_{cs}	2500 m^2
$T_{f,in,charge}$	600°C
$T_{f,in,discharge}$	25°C
$\dot{m}_{discharge}$	500 kg/s
D	0.039 m
ε	0.45
ρ_s	2869 kg/m^3
k_s	2.5 W/m-K

E-NTU and Infinite NTU simulations were undertaken over a period of one year and employed hourly Typical Meteorological Year 3 (TMY3) data associated with Daggett, California. As indicated in Table 2, charging was undertaken at a constant temperature of 600°C. In order to maintain a constant air temperature at the bed inlet, the mass flow rate of air through the open volumetric receiver, and hence the bed, must be modulated. Generally, a receiver model must be used to calculate such modulation.

A receiver model was not incorporated into the simulations of the current study. Instead, the Daggett direct normal irradiance (DNI) data were normalised according to the maximum recorded DNI value and then multiplied by a hypothetical peak charging mass flow rate of 300 kg/s. In this manner, the product of these two parameters provides a time-varying mass flow rate over the course of a year that reasonably replicates mass flow rate modulation.

In terms of control, the operational strategy of the long-duration bed was straightforward. When a non-zero mass flow rate is detected, the bed enters charge mode, with the charging air flow at 600°C and the corresponding mass flow rate. Subsequently, when a zero mass flow rate is next detected the bed enters discharge mode, with the discharging air flow at 25°C and a mass flow rate of 500 kg/s. Discharging ceases once the air temperature at the bed outlet falls below 550°C,

following which the bed enters idle mode, remaining there until the next charge cycle begins.

In both the E-NTU and Infinite NTU simulations the beds are treated as being perfectly insulated. In idle mode, thermal destratification is not accounted for.

RESULTS AND DISCUSSION
Solution Sensitivity to Spatial Grid Resolution

The numerical E-NTU solution for the air and rock temperatures was found to converge fairly rapidly on the Schumann equations solution, and at 50 nodes, the solution discrepancy was almost indistinguishable. As such, a node count of 50 was specified for all subsequent E-NTU simulations.

The convergence of the Infinite NTU numerical solution is shown in Figure 3, which displays a very important attribute associated with the Infinite NTU model. Since the Infinite NTU equation does not contain a diffusive term, the movement of the thermocline through the bed is attributable only to advection. As such, the analytical solution demonstrates that under these conditions, the thermocline has infinite gradient. Raising the numerical solution's node count demonstrates convergence to the analytical solution, although to obtain good agreement, an exorbitant node count is required. Irrespective of this requirement, a converged Infinite NTU thermocline is not physically representative of the various diffusive effects in porous flow that result in the smearing of the thermocline. The thermocline smearing observed in non-converged solutions is purely synthetic and solely attributable to numerical diffusion.
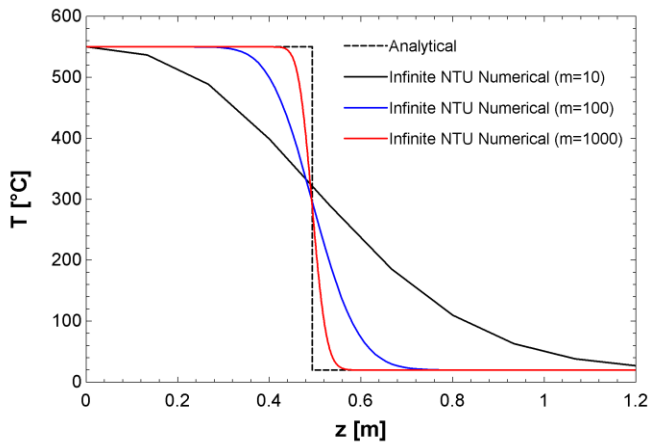

Figure 3: Analytical and numerical bed temperature predictions associated with the Infinite NTU model at 3600 s.

Since the authors are not aware of any formal procedure for establishing a node count that results in the realistic capture of thermocline gradient (by means of artificial diffusion), the following approach was used in this work. Firstly, a converged E-NTU solution to a given problem having a particular set of operational characteristics was obtained. Then, successive Infinite NTU solutions of varying node counts were derived and compared to the E-NTU solution, allowing for the node count of the Infinite NTU solution most closely resembling the E-NTU solution to be identified. For this study, this node

"tuning" was arbitrarily undertaken after one hour of charging from a uniform initial temperature.

Comparison of Numerical Predictions to Measurements

This node tuning procedure was applied to the experimental bed charging case in order to establish a suitable node count for the Infinite NTU approximations of the experimental bed charging and discharging process. For the mean bed inlet air temperature and mass flow rate calculated over the charging period, a best-fit node count of 10 was established.

Figures 4 and 5 show the air temperature histories over a two hour period associated with the experimental measurements and the E-NTU and Infinite-NTU predictions for charging and discharging, respectively. For both operations, the E-NTU solution shows better agreement with the measurements, although there is surprisingly little discrepancy between the E-NTU and Infinite NTU histories.
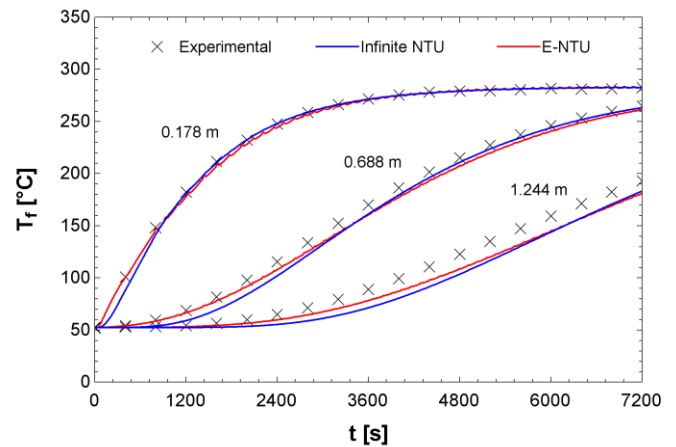

Figure 4: Comparison of 10 node Infinite NTU and 50 node E-NTU predictions with measurements during charging.
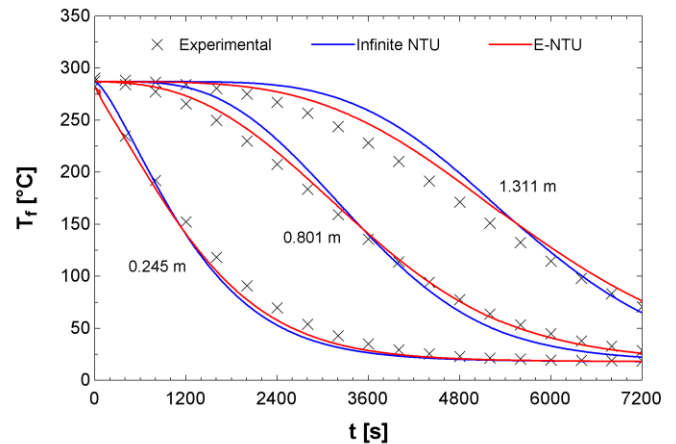

Figure 5: Comparison of 10 node Infinite NTU and 50 node E-NTU predictions with measurements during discharging.

Generally, agreement between the numerical predictions and measurements appears to be slightly better for the charging case than for the discharging case. It is also clear that the agreement between the numerical predictions and the

measurements deteriorates slightly with bed location in the flow direction. This would suggest that the air temperature prediction error is compounded with space for some reason.

Nonetheless, the authors are of the opinion that the level of agreement demonstrated in Figures 4 and 5 provides suitable validation of the numerical implementation of the E-NTU and Infinite-NTU models. Furthermore, it would appear from these figures that the Infinite NTU node tuning methodology enables a reasonable prediction of charge and discharge thermoclines, at least for one cycle.

Long-Term E-NTU and Infinite NTU Predictions

Considering the strong similarities in the bed characteristics and rock properties between the experimental and long-term numerical study beds, a nominal node count of 10 was again selected for the Infinite NTU model. Annual simulations for node counts of 5, 20 and 40 were also performed to evaluate sensitivity to grid resolution. Solution times and cumulative discharge data from the annual E-NTU and Infinite NTU simulations are shown in Table 3. A significant reduction in solution time is achieved by the Infinite NTU model when compared to the time required by the E-NTU model. The 5 node solution is achieved over ten times more rapidly, whilst the 40 node solution is obtained roughly two times faster. This is to be expected as the E-NTU model requires the coupled solution of two governing equations instead of just one.

The number of hours that the bed operates in charging mode is the same for all models (4499 h), since this value is derived directly from input data. Total discharge time, however, varies from model to model which is an important observation. Discharge mode is invoked when the charging bed mass flow is zero and if the air temperature at the bed outlet is above 550°C. Thus, variations in thermocline shape and position prediction will lead to differences in bed outlet temperature estimation at a given time, which in turn could lead to variations in the specification of bed operation mode.

Table 3: Solution times and cumulative annual results.

Parameter	E-NTU	Infinite NTU (5 nodes)
Solution time	1578.9 s	153.9 s
Discharge time	1590 h	1575 h
Discharge heat transfer	1.550×10^{15} J	1.537×10^{15} J
Parameter	**Infinite NTU (10 nodes)**	**Infinite NTU (20 nodes)**
Solution time	354.6 s	483.6 s
Discharge time	1634 h	1651 h
Discharge heat transfer	1.582×10^{15} J	1.635×10^{15} J
Parameter	**Infinite NTU (40 nodes)**	
Solution time	790.1 s	
Discharge time	1645 h	
Discharge heat transfer	1.622×10^{15} J	

Values reflected in Table 3 suggest that the discrepancy between models in this regard is appreciable. The discharge time predicted by the 5 node Infinite NTU solution, not the nominal 10 node solution, is in closest agreement with that predicted by the E-NTU model, which is considered the most accurate solution.

Variations in predictions of cumulative heat transfer between the rock and air during bed discharge are even more significant, since this parameter is closely linked to annual CSP plant energy output. The 5 node Infinite NTU model under-predicts the E-NTU value by only 0.8%, whilst the nominal 10 node model over-predicts it by 2.1%. These values are consistent with the corresponding bed discharge times. The 40 node model yields an over-estimation of discharge heat transfer of 4.6%. The fact that these cumulative data indicate that the 5 node model offers better agreement with the E-NTU model clearly brings into question the value of node-tuning an Infinite NTU model on the basis of a short-term simulation.

Cumulative data do not however indicate the level of agreement between the models at intermediate times. In this regard, Figures 6-8 display the thermoclines predicted by the E-NTU model and the 5 and 10 node Infinite NTU models at the end of months one, three and twelve of the simulations. For each model, air and rock temperature distributions are given at the start and end of the charging process.

In general, thermocline gradient is reasonably well captured by the Infinite NTU models, which surprisingly all reflect roughly equivalent gradients, despite the variation in grid resolutions. The greatest gradient discrepancy between the Infinite NTU and E-NTU predictions is indicated at the end of the first month of bed operation. This can be explained by the fact that all simulations were started at a uniform rock temperature of 25°C, thus the first month of operation results in the most significant bed temperature changes.
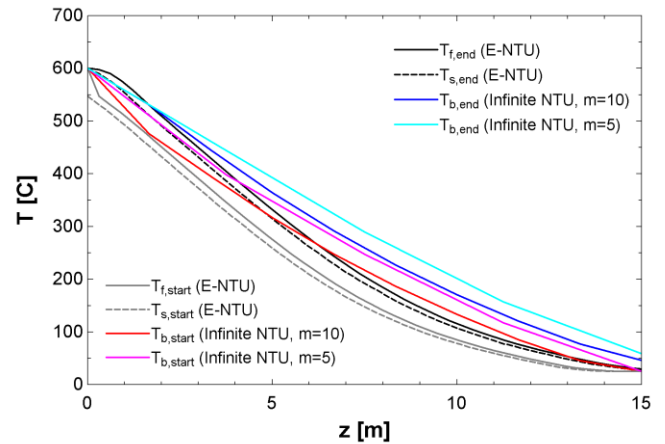

Figure 6: Predicted bed thermoclines at the start and end of charging after one month of operation.

The 10 node model appears to show better agreement with the E-NTU predictions in terms of temperature, although by the end of the year, the 5 and 10 node predictions are practically coincident. Nonetheless, the differences between the E-NTU and Infinite NTU temperature distributions would seem to be significant enough to more strongly influence the results of shorter-term simulations.

The suitability of the Infinite NTU model for long-duration CSP rock bed simulation can only be answered when the

accuracy required by, and sophistication of, the plant simulation is known. Cumulative agreement between the E-NTU and Infinite NTU models is good and thus, for preliminary simulation work, representative behaviour would be predicted provided that the spatial node count isn't arbitrarily selected. For higher-fidelity simulations, use of the Infinite NTU model could have a significant impact. Accurate thermocline resolution has a direct bearing on bed pressure loss estimation and thus the evaluation of plant parasitic losses. Accurate estimation of bed outlet temperature during discharge is required for heat recovery steam generator and steam cycle calculations. With respect to these and other considerations, the Infinite NTU model may not provide a sufficient level of accuracy. To address this, the study needs to be expanded to incorporate coupled simulation of the TES and power block.
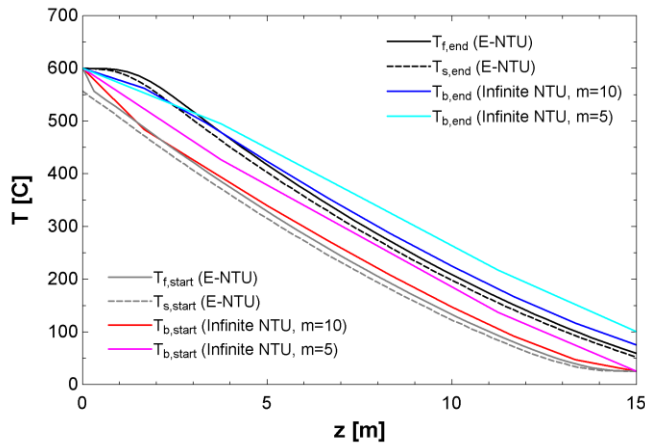

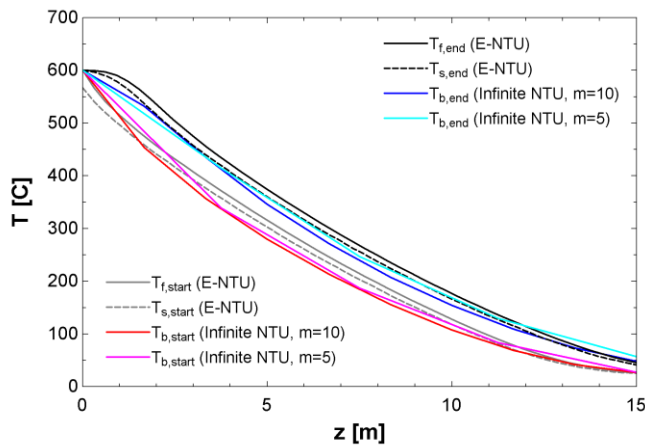**Figure 7: Predicted bed thermoclines at the start and end of charging after three months of operation.**

Figure 8: Predicted bed thermoclines at the start and end of charging after twelve months of operation.

CONCLUSION

The comparative performance of the E-NTU and Infinite NTU models in predicting the operational characteristics of rock bed thermal energy storage systems over short- and long-term time frames has been investigated. Numerical solutions to these models were compared to experimental data obtained from a rock bed test rig during charge and discharge cycles for validation purposes. Agreement between the E-NTU predictions and the measurements was reasonable, and it was found that a similar level of agreement could be achieved by the Infinite NTU model if the model's node count was qualitatively tuned.

In order to understand the behaviour of the models over a longer-term period comprising numerous charge-discharge cycles, annual performance simulations were undertaken. In addition to a 50 node E-NTU model, Infinite NTU models with 5, 10, 20 and 40 nodes were evaluated. The 5 node model showed the best agreement with the E-NTU model in terms of cumulative performance prediction, although the 10 node model appeared to generate thermoclines that better-matched the E-NTU for instantaneous comparisons. The results suggest that the Infinite NTU model can be employed in lower-fidelity long-term simulations. Coupled annual simulations of the rock bed and power block are required in order to better assess the suitability of the Infinite NTU model for this purpose.

REFERENCES
[1] Kolb, G.J., Ho, C.K., Mancini, T.R., Gary, J.A., SANDIA report: *Power tower technology roadmap and cost reduction plan*, SAND2011-2419, Sandia National Laboratories, Albuquerque, 2011.

[2] Zanganeh, G., Pedretti, A., Zavattoni, S., Barbato, M., Steinfeld, A., Packed bed thermal storage for concentrated solar power – Pilot-scale demonstration and industrial-scale design, *Solar Energy* 86: 3084 – 3098, 2012.

[3] Hughes, P.J., The design and predicted performance of Arlington House, MSc thesis, University of Wisconsin – Madison, 1975.

[4] Hughes, P.J., Klein, S.A., Close, D.J., Packed bed thermal storage models for solar air heating and cooling systems, *ASME Journal of Heat Transfer* 98: 336 – 338, 1976.

[5] Duffie, J.A., Beckmann, W.A., Solar engineering of thermal processes, Second edition, Wiley, New York, 1991.

[6] Wakao, N., Kaguei, S., Funazkri, T., Effect of fluid dispersion coefficients on particle-to-fluid heat transfer coefficients in packed beds: Correlation of Nusselt numbers, *Chemical Engineering Science* 34: 325 – 336, 1979.

[7] Schumann T.E.W., Heat transfer: a liquid flowing through a porous prism, Heat Transfer, 405–16, 1929.

[8] Allen, K., 2014, Rock bed thermal storage for concentrating solar power plants, PhD dissertation, University of Stellenbosch, 2014.

[9] Waples, D.W., Waples, J.S., A review and evaluation of specific heat capacities of rocks, minerals, and subsurface fluids. Part 1. *Natural Resources Research*, 13.2: 97 – 122, 2004.

[10] Klein, S.A., EES – Engineering Equation Solver. Version 9.486, F-Chart Software, www.fchart.com, 2013.

[11] Hiep, D.D., Transient heat transfer in porous media, MSc thesis, United States Naval Postgraduate School, Monterey, California, 1965.