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Gaussian Mixture Models as Flux Prediction Method for Central Receivers

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Abstract. Flux prediction methods are crucial to the design and operation of central receiver systems. Current methods such as the circular and elliptical (bivariate) Gaussian prediction methods are often used in field layout design and aiming strategies. For experimental or small central receiver systems, the flux profile of a single heliostat often deviates significantly from the circular and elliptical Gaussian models. Therefore a novel method of flux prediction was developed by incorporating the fitting of Gaussian mixture models onto flux profiles produced by flux measurement or ray tracing. A method was also developed to predict the Gaussian mixture model parameters of a single heliostat for a given time using image processing. Recording the predicted parameters in a database ensures that more accurate predictions are made in a shorter time frame.

INTRODUCTION

The prediction of flux across the area of a receiver is critical for heliostat field layout design, receiver and heliostat optical system design as well as optimization [1]. Numerical prediction methods such as ray tracing are used when high accuracy of the final result is required. For optimization of the flux profile, numerical methods become computationally expensive. Therefore using analytical flux prediction methods become favorable.

Flux distributions are often approximated as circular Gaussian distributions. Several aiming strategies have been developed based on circular Gaussian prediction methods [2] [3]. Although the method is justified for large heliostat fields due to the central limit theorem, low precision are often seen for small fields [4] [5].

Elliptical Gaussian models are similar to circular Gaussian models; however these models consist of two standard deviation parameters in the x- and y- dimension. The flux profile therefore adopts a rotated elliptical shape. Although this method is more accurate than the circular Gaussian model, there exist cases where a more accurate method is required.

Heliostats of small experimental fields are often manufactured such that significant errors occur on the surface of the mirror causing the flux image to deviate from the circular and elliptical Gaussian profile. Figure 1 shows a photograph of the reflective image of an experimental heliostat with large surface deviations of which the flux profile would not accurately be predicted by die circular or elliptical Gaussian method.

The Gaussian mixture model, consisting of a number of superimposed bivariate Gaussian models, is suggested to improve the accuracy of flux prediction. To better understand this method the circular and elliptical Gaussian flux prediction methods are described.

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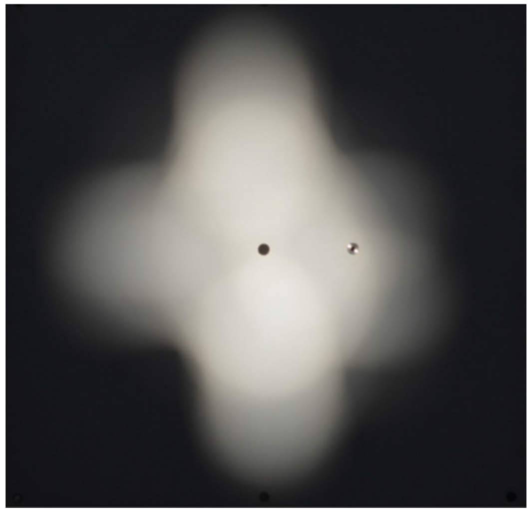

FIGURE 1. Light reflected onto a target using a single experimental heliostat

CIRCULAR GAUSSIAN APPROXIMATION METHOD

Flux prediction methods such as the Heliostat Field Layout Calculator (HFLCAL) predict the flux distribution reflected by a single heliostat as a circular Gaussian model. Each point (x,y) of the receiver has a flux value which could be calculated as

$$Flux(x,y) = \frac{P_h}{2\pi\sigma_{HF}^2} e^{\frac{-(x^2+y^2)}{2\sigma_{HF}^2}} \tag{1}$$

The power absorbed by the receiver can be determined by

$$P_h = I_D A_m f_{at} \rho cos\varphi \tag{2}$$

where I_D is the direct normal irradiance in kW/m^2, A_m the reflective area of the heliostat, cos φ the cosine of the angle between the solar rays incident on the heliostat and the heliostat normal vector, f_{at} the attenuation factor and ρ the reflectivity of the mirror. The total effective deviation σ_{HF} includes the sunshape error σ_{sun}, the beam quality error σ_{bq}, the astigmatic error σ_{ast} and the tracking error σ_t. It should however be noted that only Gaussian sunshapes should be considered when using the circular Gaussian prediction method. The total effective deviation could be defined as

$$\sigma_{HF} = \sqrt{\frac{D^2\left(\sigma_{sun}^2+\sigma_{bq}^2+\sigma_{ast}^2+\sigma_t^2\right)}{cos\varphi_{rec}}} \tag{3}$$

where D is the slant range and cos φ_{rec} the angle between the receiver normal vector and the ray reflected off the heliostat surface. The mirror beam quality is a function of the surface slope error σ_{SSE} and defined as

$$\sigma_{bq}^2 = 4\sigma_{SSE}^2 \tag{4}$$

The surface slope error is determined by iteratively varying the surface slope error until the maximum calculated flux matches that of the measured flux [6]. The measured flux could be found by means of experimentation or an accurate prediction method. The focal length is assumed to be half the length of the radius of the heliostat spherical surface and the slant range equal to the focal length. When considering astigmatic errors the tangential (h) and sagittal (w) dimension are to be considered:

$$h = D\left|\frac{d}{f} - cos\varphi\right| \tag{5}$$

$$w = D\left|\frac{d\cos\varphi}{f} - 1\right| \quad 6)$$

The astigmatic error is then determined by

$$\sigma_{ast} = \frac{\sqrt{\frac{1}{2}(h^2+w^2)}}{4D} \quad 7)$$

The final flux across the receiver surface is found by superimposing the flux distributions produced by each heliostat.

ELLIPTICAL GAUSSIAN APPROXIMATION METHOD

Each elliptical Gaussian model consists of two standard deviations and a rotation angle of the model with respect to the x-axis. The flux profile of the model can be determined using

$$f(x,y) = A_o exp\left\{-\left[\frac{\left((x-x_c)cos\theta_o+(y-y_c)sin\theta_o\right)^2}{2\sigma_a^2} + \frac{\left((x-x_c)sin\theta_o+(y-y_c)cos\theta_o\right)^2}{2\sigma_b^2}\right]\right\} \quad 8)$$

where (x_c; y_c) is the coordinate pair of the peak value, θ_o the rotated angle of the elliptical axis and σ_a and σ_b the standard deviations along the major and minor axes. Ao represents the peak flux value at point (x_c; y_c). The total power of the model can be described as

$$P_{tot} = 2\pi A_o \sigma_a \sigma_b \quad 9)$$

A more intricate description of the elliptical prediction method could be found in Guo [6]. A comparison of the elliptical Gaussian prediction models is also given. With the understanding of the fundamentals of circular and elliptical Gaussian models the Gaussian mixture model could be introduced.

GAUSSIAN MIXTURE MODEL APPROXIMATION METHOD

Accurate predicted flux profiles created through methods such as ray tracing can be significantly simplified through the fitting of Gaussian mixture models (GMM). The Gaussian mixture model can be represented by the superimposing of elliptical Gaussian models to form the final flux profile as seen in Figure 2.

Hit points determined by a ray tracer could be used for fitting the GMM as seen in the top left of Figure 3. The fitting process is based on a Monte Carlo method combined with the expectation maximization algorithm. The expectation step determines the likelihood of a fitted model. The likelihood could be described as the probability that the estimated profile is similar to the given profile. The maximization step is used to determine new parameters based on the likelihood determined in the previous step. Iteration between these two steps is implemented until convergence or the maximum amount of iterations is reached.

The first step in the fitting process is to fit a single elliptical Gaussian model on the known flux profile using the expectation maximization process as seen in the top right image of Figure 3. The Akaike Information Criterion (AIC) is then determined for the single elliptical Gaussian fitted model. This value is similar to the likelihood value as it represents the similarity between the fitted and the given data. The number of elliptical Gaussian components is then increased and another model fitted over the given data. The bottom two images of Figure 3 shows fitted models consisting of two and three elliptical Gaussian models.

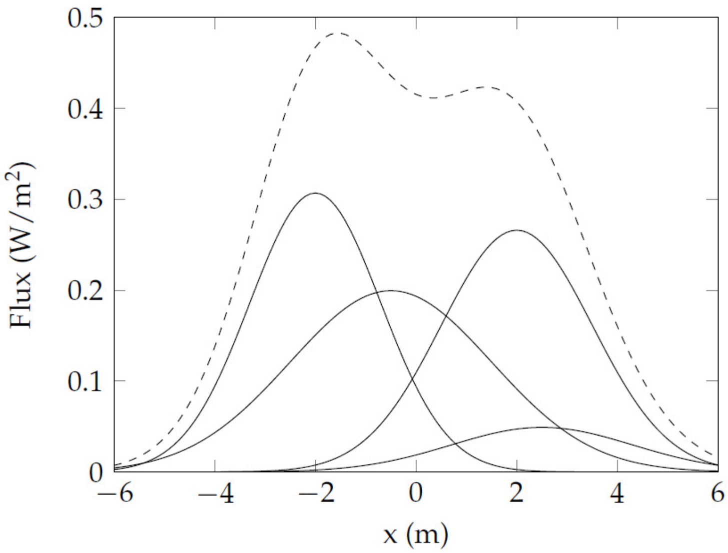

FIGURE 2. Example of superimposing of bivariate Gaussian models (Solid lines) to form a Gaussian mixture model (Dotted lines)

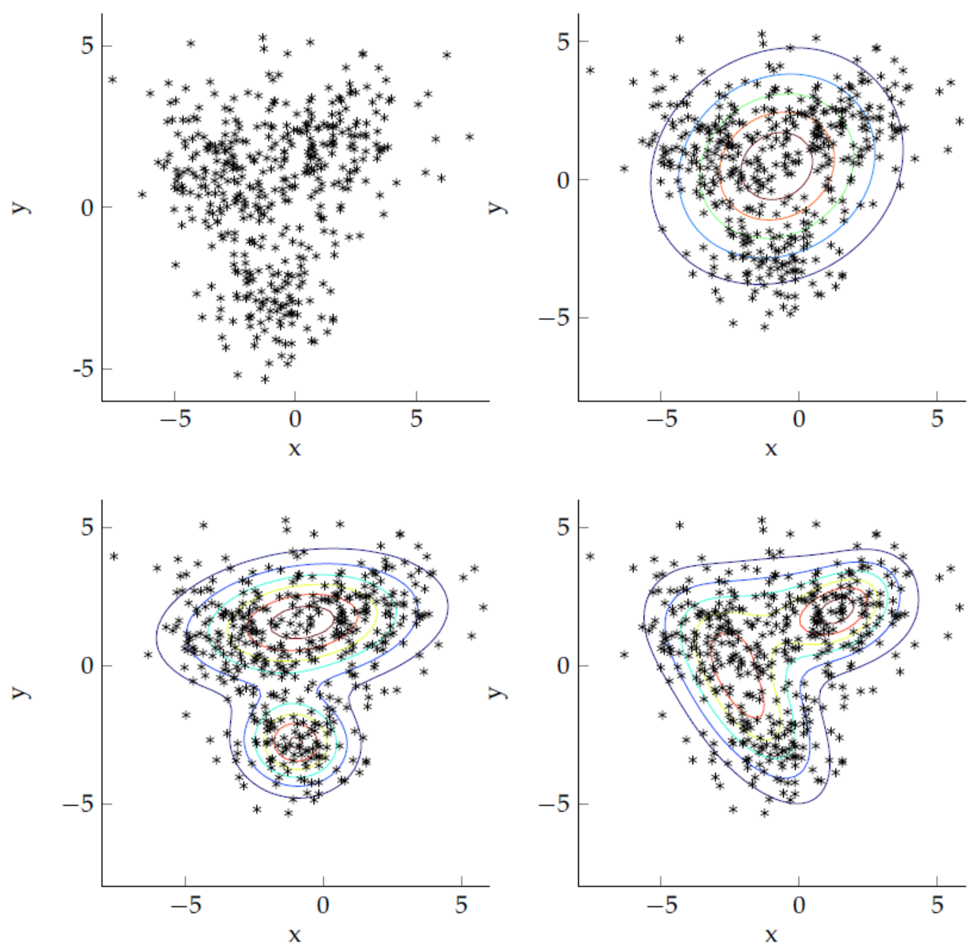

FIGURE 3. Gaussian model fitting procedure

This process is repeated for a fixed number of iterations; usually a maximum of between four and six components are sufficient. The parameters pertaining to the fitted model with the lowest AIC value is then recorded for different solar positions depending on the level of accuracy needed for the system. Two parameters are determined for each elliptical Gaussian model in the best fitted model known as the mean vector μ and the covariance matrix Σ:

$$\mu = \begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix} \qquad 10)$$

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix} \qquad 11)$$

The flux is predicted at any point (x,y) on the receiver by using

$$Flux(x,y) = \sum_{n=1}^{t} A_n e^{B_n} \qquad 12)$$

with

$$A_n = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \qquad 13)$$

and

$$B_n = \left(-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2} - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y}\right]\right) \qquad 14)$$

The final flux on the receiver is found by superimposing the interpolated flux distribution of each heliostat.

THEORETICAL RESULTS

To test the effects of using the GMM method as opposed to a circular Gaussian method (HFLCAL), an aiming strategy was run using both prediction methods. The aiming strategy is a novel strategy developed at Stellenbosch University and makes use of analytical methods of flux prediction. The Helio 40 system, consisting of 20 2m2 heliostats, was used to determine the results. Soltrace was used as the ray tracer and Matlab for the aiming strategy and image processing. The flux profiles of the heliostats are determined using a single ray tracing operation for each heliostat. The analytical function parameters (of either circular Gaussian or GMM) are determined for each map using the ray traced results. The parameters of the analytical functions are used for the aiming strategy optimization.

The image on the left of Figure 4 shows the initial flux profile for all heliostats aimed at the center of the receiver using HFLCAL. The results of the flux profile after the aiming strategy is seen on the right of Figure 4.

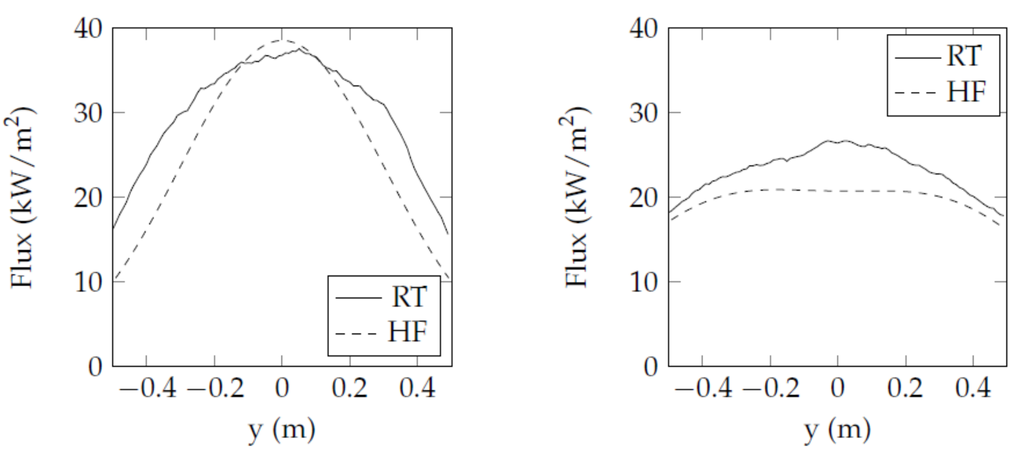

FIGURE 4. Flux profile of the ray tracer (RT) and the HFLCAL model (HF) of the Helio40 system before (left) and after optimization of the flux (right)

The flux profiles produced by the ray tracer and the HFLCAL model are significantly different. It could also be seen that because of the deviation, the aiming strategy performs poorly.

The prediction method was then changed to the GMM method. The results obtained for this prediction method is shown in Figure 5. The results of the ray tracer and GMM method are much closer to one another and the aiming strategy produced more accurate results.

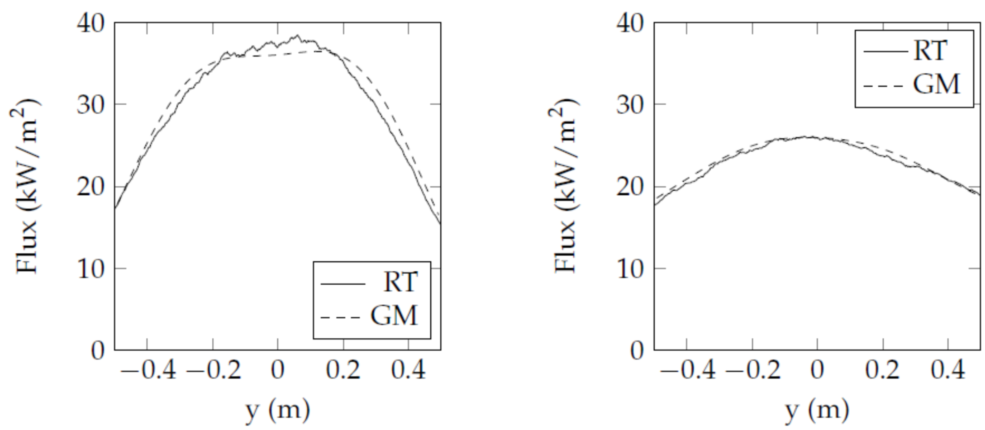

FIGURE 5. Flux profile of the ray tracer (RT) and the GMM model (GM) of the Helio40 system before (left) and after optimization of the flux (right)

Ray tracing of this system took approximately 30 seconds per iteration as opposed to the analytical functions which took roughly a 10^{th} of a second per iteration. As the ray tracer is only run once during the aiming strategy, the computational expense of the process is reduced significantly when compared to a system which only uses ray tracing. By introducing the GMM model, the accuracy of the flux profile is also improved when compared to the circular Gaussian method.

PREDICTING FLUX USING IMAGE PROCESSING

A method was developed to characterize a heliostat profile through image processing. The setup consists of a CCD camera and a Lambertian target with one or more flux sensors as seen in Figure 4. Table 1 lists the experimental instrumentation used in this setup.

TABLE 1. Experimental setup instrumentation.

Equipment	Details
Flux sensor 1	VatelCorp TG 1000-0 with AMP-15 amplifier S/N: 9627
Flux sensor 2	HukseFlux SBG01 S/N: 1596
Data acquisition unit	National Instruments C-DAQ 9181 S/N: 16615BD
SLR Camera	Nikon D5100 S/N: 50902398
Lens	70-200 mm Nikon S/N: 3678032
Neutral density filter	Kenko Pro1D 8(W) 52 mm

To start the process a photo of the reflective image of a single heliostat is taken. Care should be taken to avoid saturation of the photo such that the light intensity values are correctly represented on the digital image. At the time of taking the photo, the flux density on the center of the target as well as a third of the receiver width from the center was recorded. The DNI is also measured for scaling purposes.

Using the Matlab image processing toolbox, the image can be cropped to the correct size, squared and converted to grayscale. The brightness of each pixel will represent the intensity of the flux values across the receiver.

The intensity values are linearly scaled using the measured flux values to determine the flux distribution. The brightness or intensity of a pixel correlates approximately linearly to the flux density values. Using the measured DNI the flux profile could be scaled to represent a flux profile at 1 kW/m2.

Once the flux values across the receiver is found, the GMM fitting process could be implemented on the data. Parameters are determined for the best fitting model of the given flux profile and recorded. When the flux profile of an entire heliostat field needs to be predicted, the parameters of each heliostat for the specific time are retrieved and the calculated flux profiles superimposed on one another.

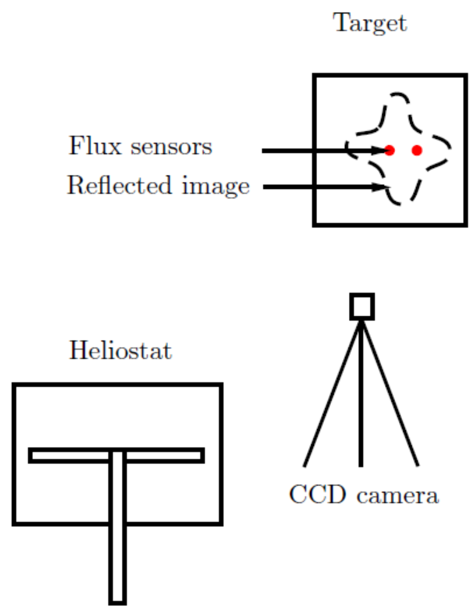

FIGURE 6. Example of experimental setup for a single heliostat

EXPERIMENTAL RESULTS

The flux profile of Figure 1 was determined using the image processing process. The image on the left of Figure 5 shows the processed flux image and the image on the right the final GMM determined for the experimental image. The GMM function made use of 9 Gaussian mixture models to produce the parameters for the predicted flux profile.

Due to the stochastic nature of the GMM function, the predicted image produces deviations from the actual flux profile. To improve this, a higher resolution image should be used.

The process was then tested using a 20 heliostat system. Heliostats were focused on the center of the target for an hour and photos taken of the total flux profiles every 10 minutes. Through comparison of the photos it was found that the flux profiles of the heliostats did not have significant change during the first 50 minutes of the experiment. It was therefore proposed that a new aiming strategy should be put in place every 30 minutes.

Further research is suggested to determine whether the method of flux prediction through image processing could be used during operation such that the flux profile of every heliostat is predicted at certain intervals during the day.

CONCLUSIONS

It was found that circular and elliptical Gaussian prediction methods often deviates significantly from realistic flux distributions for experimental and small central receiver systems. To improve the accuracy of flux prediction the Gaussian mixture model method was developed. The method provides results close to that of a ray tracer, but within a fraction of the time. A method was also developed that makes use of a photograph of the heliostat reflective image and image processing software to determine the parameters of a flux profile of a single heliostat at a given time. The parameters for each heliostat at various times can be recorded and used at a later stage during optimization of the flux profile.

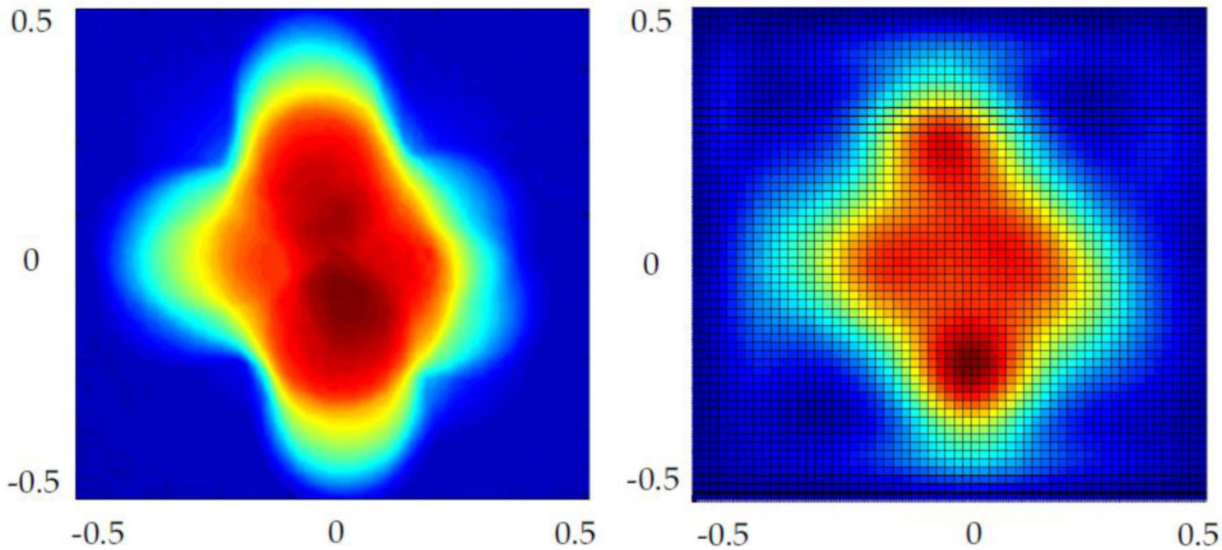

FIGURE 7. Processed photograph showing flux densities (left) and the determined GMM model (right)

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